## SHAKUNTALA DEVI

## MATHABILTC

## Awaken



## C gentus

 $5 r+1$
## Child



# MATHABILITY 

Awaken the Math Genius in Your Ch la

## SIGKUNTALA DEVI

## The Author

Born in a well-known family of Brahmin prie ts in Bangalore, Shakuntala Devi received her early $n$ mathematics from her grandfather. By the ge of iive, she was recognised as a child prodigy are an expert in complex mental arithmetic. A year te she de .onstrated talents to a large assembly of stider, sand professors at the University of Mysore.
Hailed as an authentiC her ine of our times her feats are recorded in the Guiness 900 , of World Records. She has made internaticnal headlines for out-performing and outcomputing the mit sopxisticated computers in the world.
She mairtairs that a child's curiosity and receptivity during ifacy and childhood can never be matched, and $m$ mis, as parents, nurture the young minds by offe, ing he ight learning process and motivation to de alo $\rho$ the innate strengths possessed by every child.
Th.ese days she spends most of her time giving shape to ner dreams of setting up a gigantic mathematical institute for teaching and research in maths.

## Mathability (math + ability)

"They can, because they think they can." -Virgil
Nothing is more important. In today's increasing complex and technological world the most inneta it thing you can do for your child is to nurture mathability. It is an attitude. Those yon at itat their child is poor at maths' are toi gg then colves an injustice. They are undermining th cind's future.

Mathability is a skill that tea hes a child how to think. Mathability is a st il thar develops the inherent intelligence po ent al. It enhances problem solving abilities and ana ytical focus. The methods and the techni, es are just as suitable for adults as for children. Inde. $d$ many of the methods have altered the in set even of senior executives and houser ives.

Th son th in that is often subjected to complexity, o nt sion and prejudices, Shakuntala Devi brings larity, simplicity and practicality. She corrects many of the generally held misconceptions and effectively demostrates how mathability is an acquired skill.

Nurture Mathability. Nurture Success.

## 100

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## 1

## Of Maths and Mathematicians

## Prejudice is the child os iscrance. WILLIAM H (ZLI T

The merry-g round whirled in a kaleidoscope of colo irs.) The giant wheel turned majestically on 1 s axis. Lights blazed and the loudspeake oled music and announcements alternate y. A.M. .round me voices buzzed and squealed. Thu crowd pushed, hustled and bustled. If I were to count and note down the number of times I was nudged sharply by impatient elbows I'm sure it would be a statistician's delight! The funfair was in full swing and I allowed myself to be carried from stall to stall by the sheer surge and swell of the crowd.
'Come and see the fattest woman in the world!' a voice chanted, and coaxed us towards a huge white tent. I stood and stared at the monstrous woman clad in a strange, sequinned, garish outfit. Rolls of fat bulged from her body,
making her look like an inhuman mass. There were nudges and sniggers as the group viewed this 'freak'. Saddened, I turned away and was on my way out of the tent, when two clear voices in conversation made me falter in my stride
'Poor woman! She's really a freak of ne isn't she?' said a middle-aged woman.
'She's just a fat lady,' replied the naan w th her. 'Nothing freakish about that. I think scientists, mathematicians, and artists ane the or gina freaks. Mental freaks!'

There was something - lib rate about his tone that made me realise hat h, had recognised me!

## Maths: Mixed reacion \%, strong emotions

I have, by no $v$, become used to the varied reactions th at greet me. Sometimes it's unnerving. Even witt m. considerable mental ability with numbors, 1 still not clear on one point: whether ben $g$, mathematician has more pluses or m unes! Perhaps every profession has its share o. strange reactions. I know a doctor who, when on vacation, refrains from prefixing his name with 'Dr'. 'The moment an acquaintance comes to know I'm doctor, I have to listen to a list of all his aches and pains!' he says with a comical lift of his eyebrow. 'And there goes my holiday!'

A writer friend has quite another tale to tell. A brigadier moved next door to him. My friend being of a friendly disposition, invited him to his house for a neighbourly drink.
'What do you do?' asked the brigadier, as they toasted their budding friendship.
'I write,' replied my friend. 'I'm a writer.'
'I see,' said the good brigadier, puzzled. 'But what do you do? Don't you have a profession?'

You'd think that in this world where the sciences are respected, a mathematician would be considered the creme de la creme of societron N that I have never experienced this feeling. I F ve. When people realise I'm a mathemat ian. I \%o see a spark of respect. But not alyays. A mathematician's image, I'm A' ra d, i; osged by a hotchpotch of numerals symb 1 , and fractions. A computer instead of a cers on ... dry, dusty, unattractive ... coldly linic..., who peers at the world through grey-tinte glasses... dressed like a tramp.

## Mathematicians: Talented, gifted, but . . .

A little git once, remarked, 'Why! You are nice!'
'Th n'. You,' I said amused.'
Is mom told me I should behave myself and in t/act silly and giggly with you,' she went on with childlike candour. 'But she was wrong! You laughed at all my jokes!'

How refreshing it was to hear that! The little girl was right. Mathematicians can laugh. How I wish that adults were as open-minded as that child. Most people slot mathematicians into a pigeonhole of preconceived ideas. They assume that I want to be always addressed as 'madam', that I don't know a thing about life outside the pages of my numerological epistles! I've met people who, when introduced to me, say 'Hello' and
then fidget uneasily. I can see what is going on in their minds: 'What do $I$ say to a mathematician?' It is as if I am a being from another planet who doesn't even talk the same language!

I've had such remarks thrown at me:
'I'm a "dummy" at maths!'
'Could you teach my son some triel's to na.e him pass his maths exam?'
'God, you're lucky! You nu t a woys know exactly how much chang you ave to get when you buy something!'
'Beyond knowing vo prus two makes four, I'm zero!'
'Do you hav any time for hobbies?'
'Don't year ver get bored?'
Whentive get to know me, people are surprise that I'm human too! That I have a sense of bim u. And that I can talk about food, family, mt sic films ...

A mathematician is usually known by two nitials - BB: Brilliant and Boring! The 'boring' is part of a self-defence mechanism that stems from the other person's own feelings of inferiority. It is a strange social phenomenon. For some inexplicable reason, mathematics is seen as an exalted cloud on which only a chosen few reside. What it lacks is glamour.

For this reason, the mathematician finds himself or herself in a no-win situation. Talented, gifted, great for a maths quiz show perhaps, but not interesting enough to have a cup of tea with!

I suppose it has something to do with the aura surrounding the persona. A film personality has an aura of glamour and glitter. A saint has an aura of spirituality and humanity. Both have an 'otherworldly' halo attached to them. But the mathematician's aura is unfortunately seen as a combination of musty figures and wring calculations! How often have you hered a mathematician being interviewed for a nagazine article? I think the weakness lies ine lact that there has been almost no at empt .0 make mathematics 'people-orie ted' $\mathrm{I}_{\mathrm{t}}$ is, viewed exactly as it is defined in the dictio ary: an abstract science of space and numbers. Where are the emotions, the culture, the human ing s, the highs and lows, the conflicts, and the drama that draw people to a subject? Wherd are the anecdotes that give flesh and brood oo the most abstract of theories or thougits?

## Fin 'im ressions:

## Nv. q. ... mathematics, ugh ... mathematics

I asked a friend about her earliest recollections of mathematics. I'm quoting her verbatim because I think her words provide a piercing insight into the misconceptions about maths imbibed in childhood:
'Initially, maths was an easy affair. My mother would place first one apple in front of me and then another. So I knew that one plus one equalled two, and so on. I graduated to fractions. She'd cut the apple, and I learnt about "half" and "quarter". But my problems with maths began in school. My father now entered the picture. He
insisted that I get full marks in maths. This jolted me. Suddenly, the subject I had enjoyed became a burden. My maths teacher was a stern old man, always dressed in an ill-fitting, rumpled, greychecked jacket and brown trousers. He'd call eack of us to the front of the class to work out a sur on the blackboard. If we did it right, he' sa "I knew you were a clever girl." But if we not it wrong, he'd say, "Bad girl!" and ba. ish as .o the back of the class. I'd approach the bir board every day with the terrible for of bt ing rublicly labelled "bad". It was a humin tho experience.'

The negative approach of a parent or teacher with high expectation withsut the element of fun, drives the child at ay from what is really an interesting subject.

The first inpres ion remains with the child even as he or sh. matures into adulthood. When the child is fer a new titbit, her first reaction is to $s_{1}$ it it out because of the foreign taste. Someth e, the 'bad taste' remains. So it is with m. th matics. It has left a bad taste in the mouths if. majority of people. The almost paranoic distaste of maths is transferred to the mathematician. But I hope I can change that image.

Mathematics is a fascinating subject. In the next chapter, I've attempted to lift the iron curtain, metaphorically speaking, to give you a glimpse into the more human aspect of it. With a better understanding and a more human approach, I am sure you can influence yourself and your child to change that 'sickening, mushy' stuff into something that is enjoyable, delicious and crunchy. By the end of this book, I hope you will be saying: 'Mmm ... mathematics!'

## 2

## Maths in Everyday Lio

> Nothing has changed but my anmitit Everything has ern.ed. ANTHONY DEMTL

I've met moe, eople who profess a hatred for maths tha those who are indifferent to it. For some re son the subject is like a red rag in front of them. Sucion graces are dropped, as such maths hat ${ }^{\text {er }}$ tell me with a shiver of distaste, 'Maths 1. bo ing.' Initially, I found it strange. I couldn't magine walking up to a history professor, for instance, and telling him: 'History is boring.' Not that I find any subject tedious, but the point I am trying to make is that social etiquette demands a certain diplomacy. Maths seems to strip away that veneer to show the snarl beneath. It puzzled me, until I attended an advertising workshop.

Smartly clad, articulate executives spoke about 'upmarket advertising', 'brand loyalty', 'marketleadership', and so on. They projected slides to demonstrate or prove a certain point, or even to
create an interest. One gentleman spoke at length about marketing a soap to a specified target audience. Communication was touched upon. There was a discussion on products endorsed by celebrities. I went home with all the advertisirg buzzwords running through my brain.

The idea hit me at around midnight. Now I knew why maths was so unpopular. Or cour e! It was so simple! Why hadn't I realiz it it arlier? Obviously, the subject had nver been warketed! It was treated as a 'product'; it as of a high standard but it had no glo sy ackaging, nothing to recommend it. It h. d exc, llent salient features built into it, but milions of people all over the world were unaware $£$ it. Mathematicians and teachers had raa no attempt to communicate these excitig q, alities. Literature, on the other hand, is cel-marketer'. Its essence lies in its commuvirative abilities. Maths needs to be maket d, Siace it is taught using numerals and sy nb ls, it is not so much a foreign language,
an alien one.
The language of symbols is alien to us since we don't use it in our everyday speech. We don't use it in our daily communication, except perhaps in a specific context.

## The maths alien

This 'alien' feeling comes through clearly when a person meets a mathematician. The 'numbers person' is viewed as a different species. And since maths has the label of 'high intelligence' attached to it, the other person feels inferior. Predictably,
he or she turns away, or is hostile. He cannot comprehend that 'this-person-who-deals-and-probably-thinks-in-symbols', is also a living, breathing, emotional human being with normal desires and aspirations. That the mathematician talks the same language, eats the same food, tennis, and listens to music, is ignored

For many, 'maths-alienness' begins ...Do. we. A budding actor told me, 'I remember I had to sit at this huge dining table a ome. My grandfather, who was a wizat + patns, would check my homework. If 1 It de a mistake, he'd thunder at me. He co, sidert yue a failure because both he and my fatner wad been whiz-kids. I was not seen as a chip ot the two old blocks!' The human mird a funny thing. Negative impressions ma de in childhood seem to remain imprinted on par minds forever. Parents with high espetanons come down heavily on their childan, d. A ing them into a corner. Sometimes sibn ing iviries are triggered off by obvious and III (ai) comparisons.

If you have had similar experiences, take them out and examine them. More often than not, you will find that your own maths-alienness arises not out of your 'natural' hatred for the subject, but from your disgust or rebellion against an authoritarian adult. If you can recognise that, you will already be on a new, exciting road strewn with fresh possibilities and opportunities. You may shrug and say, 'I've done without maths so far, I can do without it forever, thank you!' But look at it this way: why would you want to shut your mind to anything?

As natural as breathing
What I want you to do is to overcome your feeling of maths-alienness, if you have it. I want you to enjoy every moment of your life with maths. Maths may or may not play a large part in your die, but it is essential in every field. Whether ycu realise it consciously or not, you are breat ing, living, even eating maths.

After all, you have to programmit 0 Ir day: wake up to an alarm, fix $\mathbf{D}$ ee kfast, watch the nine o'clock bus. When/wo 0 rcise, you count the sets, check your pu e late. You may be measuring your daily calorie intake. When you eat out, you glance a thy right-hand side of the menu card towavoid prarring beyond your budget. You scan bills ©or accuracy before paying them. You balance you chequebook every week. When you liste music, your mind absorbs the rhyth mil (rad: mathematical) beat. You time your ove $t_{1}$ e , essure cooker. Statistics reported in ne spapers interest you - like the number of run scored by Sachin Tendulkar or Imelda Marcos' shoe collection! Or that in 1990, Evander Holyfield, the boxer, was the highest paid athlete in the world - his income totalling $\$ 60.5$ million. Then, you may hear on the news that a 105 -yearold man still lives on, hale and hearty. You may listen to the weather report to know at what temperature you are sweltering in!

By doing all this you are already tuned in to a fascinating mathematical world. It is as natural as breathing. You are already enjoying a major portion of it; why not enjoy all of it?

Your competitive edge
I've talked about everyday maths in your life. But even professionally, you may find that taking up a maths-oriented course gives you an edge over your colleague for a promotion. Or you may fird that reading and analysing graphs gives rou a better insight into your area of work EV $n$ if maths doesn't come directly into your protessinn, you could land a job because of your ine st in it!

Vijay was a chemist at a pherreceutical factory for several years, Slowly he began to feel he was in a rut. He scan ed the wanted ads in the papers everyday. A conpany specialising in holding chemical ex ib, ions, called him for an interview. Durin the ourse of the interview, the proprietor ask hrijay about his hobbies. It so happened that jay was an amateur astronomer. Casuall, he -entioned it as one of his hobbies.


The proprietor was electrified. He happened to be one himself too! The same day, Vijay walked out with-his letter of appointment as a manager in the company.

So, you see, you never know what opportinities await you in this great, wide world! wn not make the best of them? You should rot stop yourself from taking an excellent inb shint ${ }^{1} y$ because one of your duties may in olye little maths. The job may be perfact in very way. It could be the making of you. It could be 'The Dream Job' that would in II, O, C interest, your aspirations, your am itions It may have a fabulous salary and unima inoble perks attached to it. Everything abouf it nay be what you've hoped for, except for that one little clause.

There aro $t$ so scenarios: one, you chicken out, in which calse you've lost a great opportunity; two, you ake it, but dread the maths part of it. Ent el why you are miserable. You curse y,ur elt, your job, your boss, your colleagues.

Jut let me paint a third scenario for you. You decide to take the job. You are determined to tackle every duty with verve. You tell yourself, 'I'm going to enjoy totting up those figures!' or words to that effect. As you get deeper into it, your optimistic approach pays dividends. You find you are enjoying it. You realise that you never knew you had this talented side to you. You are more confident, more self-assured. You know now that you can tackle anything. You've joined the league of when-the-going-gets-tough-the-tough-get-going!

## You are the master

It is extremely important not to allow a feeling of man-made maths-alienness shut off channels to wider vistas. For starters, change your outlook about maths. See it for what it is. Don't let 7 negative parental attitude influence your life oda Forget phrases like:
'I've got no head for figures.'
'I hate maths.'
'I can't stand the thought o. balancing my chequebook!'
Just as you burdle up ond refuse and throw it in the trash bin, so sh yid you with your old, outdated fears. You are going to start afresh. No, you are never too sld to learn a new thought process. Old Ma * Maths is not $2 \mathrm{~m}_{\mathrm{c}}^{-2}$. He is a friend Tikna tried and cest d orse he is always at yoar beck and call, I. ary to be harnessed. It is up to you to take up the reins and guide him o wherever you wan
 him to go. Remember, he is not your master. You are the one in the saddle. You call the shots.

## 3

# Origin of Maths: The 

## Dynamic Numenals

All acts performod ise world begin in the in agiration.
BARBARA GR, 乙ZUTI HARRISON

Who ione greatest, most brilliant math matician in the world? You won't find hir in the person-of-the-year list. You won't see his or her name in the hallowed halls of fame. The eres I am referring to is: Mother Nature. , re you surprised? And well may you be. After ah, you have normally viewed nature as the great cosmic force which fills your world with beauty, and, sometimes, disasters. But I am not talking about flora and fauna. The breathtaking visions that you see all around you, the artistic twirls, the glowing colours, are all manifestations of a natural law or plan laid out by nature with mathematical precision!

It is nature's blueprint that charts the paths of the stars and planets. Such is her precision, that astronomers can know in advance the exact
location of these heavenly bodies at a given time. You can see the sun rising in the east and setting in the west as though it has been programmed. Similarly, seasons follow one another in perfect cycles.

And you, nature's child, ride on this mathmagical current - ebbing, adygrin\%, flowing with it. So, you see, mathematio in res through our very system. It is silen never seen at work, and cannot be perceivea th-ough our five senses. That is the reas on wa he been unable to recognise it. it is t.o abstract, too intangible. When you watch a unrise or a sunset, you are only seeing the result of this great intangible power.

## The Indian ${ }^{\mathrm{N}} \mathrm{u}_{\mathrm{n}} \cdot \mathrm{eral}$ System

I think the ic of numbers permeated into our ancesto 5 newhere in their minds, a seed was sow a a b gan to flourish. Slowly, the ancient saces of India worked on and evolved a numeral s. stem. Today, we have a name for it - the Decimal System. But for our great forefathers, it was a creative endeavour, a labour of love. Their thought process evolved on the basis that with only ten symbols - the ten fingers on their hands - they should be able to represent any number. It sounds so simple. It is, but only a brilliant brain could have developed it thus. The ten symbols are our basis for calculation even today. We call them numbers or digits. We write or read them as: $1,2,3,4,5,6,7,8,9,0$. These ten symbols stand for or are pronounced as: one, two, three,
four, five, six, seven, eight, nine, and zero, respectively

The sages didn't stop there. They took on the bigger challenge of positioning or placing these symbols. The far right position in any numer 1 became the unit. Moving left, the next pos became ten. That is how we have ones, ten; hundreds, thousands, and so on. When ${ }^{n}$ ite these positions, they look like this:
1 (unit)
10
100
10.0

The most exciting part of this positioning system is that is is so universal. Take a more complex number like 5081. How do you break it down?
$(\underline{5} \times 1000)+(\underline{0} \times 100)+(\underline{8} \times 10)+(\underline{1} \times 1)=5081$.
rociry, when we say, 'The price of this item is fiv 'housand and eighty-one rupees,' it rolls -ff ur tongues so easily. But it was our ancestors wno made it possible for us to express ourselves in this way. We are able to put a value to the item and do not need to rely on the old, primitive barter system.

You can see this system at work graphically too. For example, take the figure of 37 and translate it into dots:

| $\cdots \cdots \cdots \cdots$ | $=(10 \times 1)$ dots |
| ---: | :--- |
| $\cdots \cdots \cdots$ | $=(10 \times 1)$ dots |
| $\cdots \cdots \cdots \cdots$ | $=(10 \times 1)$ dots |
| $\cdots \cdots \cdots \cdot$ | $=(7 \times 1)$ dots |

You will notice that each line or group represents ten dots, except for the last one which has seven single dots. If you write it down, it would be:

$$
(3 \times 10)+(7 \times 1)=37
$$

You will also find that there is an emrha is on tens. How did our forefathers come up with this magic figure of ten? As mentign d earl, er, it is most likely that they relied on, or spund a measure from the most fle ible purts of their bodies - the ten fingers!

It is certainly food for ti pught, isn't it? Imagine the sheer ingenuity or man tnat made him work out an entire mathe nat zal system from a part of his anatomyi kven today, you see people counting on their ingers.

It is in por ant to understand the full impact of this thpught process because only then will you Mo 10 understand 'Old Man Mathematics'. He vas Eorn out of you, and you have him at vo $r$ fingertips!

Sublimity and simplicity are the essence of our Indian civilization, of the wisdom and sagacity of our ancestors. Perhaps that is why it is so symbolic and fitting that the seeds of a singularly phenomenal science of mathematics should have sprung from India - a silent but dynamic science that stormed the world with its great possibilities.

## The Arabic Numeral System

How fortunate for the human race that in those days there were no travel restrictions. Traders,
merchants, and explorers crossed countries and sailed the vast oceans freely, bearing not only goods but also knowledge across borders. And so it came to pass that the Arabs learnt about the Indian Numeral System and adopted it is their own. The great chain of communic? continued and reached the borders of Etrop

## The Story of Zero

It is believed that the Hindu- a a symbols for numbers have been usea asm $r$ ys five or six centuries before the irth of Chist. In the earliest stages, however, the Hindu-Arabic system of number notation dids ot contain a symbol for zero. Without the zer the system was not of much use. The earliest $k$ own use of the Hindu-Arabic zero occurs in an Indian inscription dated 876. B.C. And the inwsputable superiority of the HinduArbir sum over all others is a consequence of tro acing the zero concept and symbol.
The word 'Cipher' means zero. 'Cipher', comes from the Arabic 'Sifr' and our word zero is derived from this word.

1. Any number plus zero equals the number.
2. Any number minus zero equals the number.
3. Zero minus any number equals the negative of the number.
4. Any number times zero equals zero.
5. Zero divided by any number except zero equals zero.
6. The operation of dividing by zero is not defined and is not permitted.

The earliest known 'maths merchants', instrumental in transmitting the Indian Numeral System from Arab sources, were Robert of Chester and Abelard of France in the twelfth century. The exciting new invention was well received in Europe, and was duly christened the Arab Numeral System. Later, when it was discevered that it was of Indian origin, it came to le Lm wn as the Hindu-Arab system.

## Evolution of Roman Number.

The world of mathema io $u$,bles over with interesting stories. Ut fortur tely, our present day teachers rarely discuss the interesting human face of this wonderful scier efor example, the ancient Romans had the own method. They converted letters to renre ent numbers, giving the Roman system its owr Identity. Thus, I stood for one; $V$ stoor 1 or ilve; $X$ stood for ten. In short:

$$
\begin{aligned}
\mathrm{I} & =1 \\
\mathrm{~V} & =5 \\
\mathrm{X} & =10
\end{aligned}
$$

To give meaning to these alphabets, they devised a few simple, general rules:

1. Repeating a letter doubles its value: $X X=10+10=20$
2. A letter placed after one of greater value adds to its value: $\mathrm{VI}=5+1=6$.
3. A letter placed before one of greater value subtracts from its value: IV =5-1=4.
4. A dash over a numeral multiplies its value by a thousand: $\overline{\mathrm{X}}=10 \times 1000=10,000$.

How painstakingly, but logically, must the ancient Roman intellectuals have worked out this system! And how they must have rejoiced when they charted it to the end!

Thanks to the brilliance of Indian and Roman mathematicians, we have inherited two nu systems. However, it is the Indian metho' that has prevailed. Interestingly, the Roman mo als are like a shorthand translation of the Indo-Arab numerals as their value increases, s, Ir a can see for yourself.

Many people have told 1 e that they are nonplussed by high value ' rab ic numeral. A highflying business tycoon once :old me humorously, 'I have to be careful wh n I say "billion".' In the USA and France, a vilion equals a thousand million, wheres in england, a billion is a millionmillion! Upto a point, India has her own

|  | F-A | I/A | A) and $R$ | Rom I/A | $\begin{gathered} \text { an }(\mathrm{R}) \\ R \end{gathered}$ | $\begin{gathered} \text { Numer } \\ I / A \end{gathered}$ | R |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 11 | XI | 30 | XXX | 5000 | $\overline{\mathrm{V}}$ |
|  | II | 12 | XII | 40 | XL | 10,000 | $\overline{\mathrm{x}}$ |
| 3 | III | 13 | XIII | 50 | L | 50,000 | $\overline{\mathrm{L}}$ |
| 4 | IV | 14 | XIV | 90 | XC | 100,000 | $\overline{\mathrm{C}}$ |
| 5 |  | 15 | XV | 100 | C | 500,000 | $\overline{\mathrm{D}}$ |
| 6 |  | 16 | XVI | 200 | CC | ,000,000 | $\bar{M}$ |
| 7 |  | 17 | XVII | 400 | CD |  |  |
| 8 | VIII | 18 | XVIII | 500 | D |  |  |
| 9 |  | 19 | XIX | 900 | CM |  |  |
| 10 | X | 20 | XX | 1,000 | M |  |  |

nomenclatures. For your entertainment, I've given, a table of higher Indian-Arabic numerals with their American and European counterparts:


So, the next time you want to let off steam, try, 'I've told you a quadrillion times not to do this!' It is sure to get a reaction nonillion times over!

## The Imperial System

The evolution and branching out of mathematics could be a cartoonist's delight. Yes, indeed, it has its own idiosyncracies! The primitive Anglo-Saxons had a rough and ready reference for measurir g - their hands! But for obvious reasons, $\mathrm{tr}_{\mathrm{i}}$ 's method was undependable. And it woyiun be wrong to say that one man's inch might he andth se man's foot! But they managed son ohov until they derived the Imperial System.

Though the new Imperial sysum was based on the old Anglo-Saxon ine, àn attempt was made to make it more unifom and exact in the British Empire. After in bone-cracking, the royal mathematiciars bocided that the inch was the length of the suckle of the thumb!

King Edgar $\mathrm{r}_{\mathrm{t}}$ ast have grumbled, 'Give them an inch ha.th ey take a yard!' The next thing he knen way that the yard was the distance from the ip if (ic royal nose to the tip of his majestic m. a a finger as he held his arm and hand - ts, retched!

Perhaps the mile proved to be a 'milestone' around their necks! Here, they decided to borrow from the Roman legionaries. The Roman milli was one thousand paces which, when measured, proved to be about 1618 yards. But paces varied, so the mile was eventually standardised at 1760 yards - a nice round figure.

Satisfied that they had taken the first step in the right direction, the math-men turned their minds to a larger area. I can quite imagine the Englishman standing with his lips pursed, gazing
at the oxen bearing the yoke on their muscular shoulders, as they ploughed the land. As the sun began to set, he might have exclaimed, 'By Jove, l've got it!' And the acre was furrowed out by measuring the amount of land ploughed by the oxen in a day!

You have to admire the mathematio ans immense observational powers and creaing in bringing some order to the Impe ia System. English-speaking countries, inclare ghe USA, naturally adopted it.

## The Metric System: ©ustem tic $\mathcal{E}$ deliberate

France, however, ev lve the Metric System in a more systematio, or dy liberate manner. The metre was equalled $\omega$ onten millionth of a quadrant of the earth's mer dian. Napoleon gave it his stamp of approwi ard it was adopted by France and introdued other European countries.

Of purse, eventually, a more sophisticated apirouch was adopted and standardised. To give y. 1 an example, the metre is now defined as 'the length of the path travelled by light in vacuum during a time interval of $1 / 299,792,458$ of a second.' Try that one out in your next family quiz contest!

## Music of Maths

I've left the choicest bit to be savoured last. Mathematics, to use contemporary terminology, proved to be a user-friendly science. It led to the development of astronomy, physics, and other sciences. But if you think that it has been used
only in the world of science, you will be surprised. Maths has dipped its talented fingers into the arts too. Western and Eastern music have their roots in mathematics.

On his way home one day, the gre t mathematician Pythagoras suddenly stopped an a swivelled around. His keen ear had gaug't a delightful ring coming from a shop - ing th at seemed to have a certain rhythm a d barmony to it. Eager to find the sours? fe pyed inside, and what do you think he saw? Masicians? No. He saw five blacksmiths wela $n$ g their hammers on the anvil! As $m$ wa.ched and listened entranced, he obserea that each hammer was of a different weight, a d made a different sound as it thumped gan the anvil. He soon realized that the heavy, ammer produced a lower note than the iobter one. He frowned, feeling instincti ely that there was something wrong: one Gan me was not quite in sync with the others. It way slightly off-key, he felt. Tentatively he e olained this to the blacksmiths and asked if he zould borrow their hammers for an experiment. The humble men were surprised but agreed readily.

Back home, he weighed the tools individually. Each hammer yielded a different weight in a certain proportion to the next one, except for the 'off-key' hammer. When he corrected it to suit his calculations, he found that the rings were now in synchronization! By constant experimentation, he soon devised a musical scale!

An Indian classicist once told me, 'Talas are simple arithmetic!' Talas are rhythmic cycles of
a group of beats. For example, the teentaal has 16 beats divided into four groups each. It goes:
na-dhin-dhin-na/na-dhin-dhin-na/na-dhin-dhin-na/na-dhin-dhin-na.

$$
1-2-3-4 / 5-6-7-8 / 9-10-11-12 / 13-14-15-16
$$

Mathematically, one could write it as.

$$
4+4+4+4=16 \text { or } 4 \times 4=
$$

One school of Indian music caled Farnatic even harnesses fractions! E en uany, what we hear is a measured mix of sou $a_{0}$, which, when emphasised, slowed down, or a celerated produce pleasing rhythmic mus ! It w wld be too sweeping a statement to say nu ic is maths or maths is music. But without a neasured discipline, there would be no music - Western or Eastern - only a cacophong of sounds.

I hop vave succeeded in arousing your interect in the colourful culture of mathematics. Belk ve ne. $\%$ is a wonderful world. In some ways, it ine learning a new language. It is challenging, s. mulating and, for that very reason, fascinating. bike music, it has a wide range - from the minuscule atom to the vast skies and beyond. It has permutations and combinations that add to its dimension. Maths is a great subject. Don't knock it, just try it!

## 4

## Maths is Essential For Success!

To know what has to be do, e, men do it, comprises the whole, hilosophy of life. SIR CNIL IAM voLER

Dictators an viewed with distaste, hatred, suspicion ).? yet parents themselves, more often th an nor, are dictators in their own homes. They uri their children's faces in the direction they feel is right for them. The strangest thing bo. $t$ this self-defeating exercise is that the direction is decided not on the basis of the aptitude of the child, but by the inclination of the parents.

The dictator uses his trained army to wield his power. Parents use their image as role models for the child, and the tiresome axiom of 'father knows best' to wield their power. Smiles and gentle words make this attitude no less objectionable. They only make it more palatable as they brainwash young minds.

Parents and relatives obviously influence a growing child's mind. Strangely enough, even teachers whose very profession consists of teaching maths, can make as big a mess as the parents. I find this astonishing, because teaching by ito very nature should have good communicatio blended into it. The fact is that roo 1 mathematicians are rarely good communima rs. I think maths courses should include the fine art of communication as a subject. This is very important as it would have a ro itive Enpact on our future generations. This a, both girls and boys would grow up wilh well-rounded personalities. At preser $t$ it is a one-dimensional mental growth, whicher way you look at it.


Maths should be enjoyed and understood, not shunned totally or taken as bitter medicine.

Some people I have spoken to blamed the personality of the maths teacher for having had a negative impact on them. For example, Rima, a homemaker, told me, 'My maths teacher an awkward, bony, bespectacled spinstex clled Miss Nair. She had a funny walk, as thougs se had a hernia. I always had the far that if I became proficient in maths, I'd becu mike her.'

Rima's outlook was imma und illogical. In fact, I feel sympathetic towa ros Miss Nair. She didn't fit the stereoty, e fen inine model that has been pushed down ou theoats by society. That she taught a 'máscu 'ine' subject like maths probably adaed ont. Rima confessed that her mother and whe vould privately poke fun at this teacher. I d'dn't want to hurt Rima's feelings or I'd have told ner that it was her mother, not Miss Nair, he was to blame. It was her mother who wh's erpetating this narrow vision of a woman's olt in society.

Another young woman called Lily had a more amusing anecdote to relate. She described her maths teacher as an old man whose clothes always smelt of mothballs. 'I related maths to that musty smell,' she concluded, 'and couldn't bear it.'

It is strange how unimportant, isolated little details or incidents can make or mar a child's interest in maths.

I hope through my book to influence more and more people to consider maths to be a friend,

equally accessible to bo women and men. We are all blessed with byins and nobody is less intelligent thay the other in sphere, including mathematics.

In so cultures while boys are often enco ag d 19 study mathematics, and expected to $n$ aste. $\%$ girls are subtly, diverted away from t. inder the mistaken belief that it is not for th, in. Infact, speaking symbolically, I'd say that the first mathematician was a woman - Eve. Yes, Eve. She plucked one apple from the tree of knowledge and decided to go halves with Adam. The symbolism lies not only in the mathematical terms I've used, but in the larger context too. She plucked it from the tree of knowledge. She displayed a curiosity that is the very basis for imbibing any knowledge.

## Maths: A success multiplier

One very. exciting aspect of mathematics is its ever-evolving nature even as its foundation remains firm as a rock. It's like a tree with strong roots, whose branches spread wider and wid r.
'Mathability', today, is not just confinea to engineering, accounting, geology, com uter programming or such technical fip1/45. 1. as percolated down to various manag mert fields as well. Earlier, an executive in ga n er / export firm may have been chosen bec. usoof his pleasant manners, his ability th hale clients, his knowledge of fabric ana fashions. Today, in addition, he is erve ted to handle 'data', 'statistics', and 'irrestigative graphs' do 'quantitative naryses', and 'target planning'.

A cand date, who presents himself as ideal for a particun $r$, has to have these technical skills. It is no. erough that he is able to read export jou nay ord magazines to keep up with the tre do. He must be comfortable reading figures a d 'thinking mathematically' as well.

With increasing competition in every field, analytical data is fast becoming as common as literature. If that sounds ridiculous, I assure you that even literacy was initially confined to a few until it gained universality. Mathematical-literacy has an important role in this high-tech age, and we must get our act together if we do not want to be left behind.

There is maths in everything. Maths is both the mother and child of all science and art. It is the
heartbeat of learning, the alphabet of one school of knowledge. It is the foundation of all order. Even the simple 'two plus two equals four' formula gives us an exactness and yardstick. Without order, all would be chaos. Whether is our heartbeats, words or chants, musi mantras, maths permeates them all, giving thein order, direction, and the space to bren-1 ut deeper and deeper into knowledge

In a way, maths is akin to the Fh, Lastern or Chinese calligraphy. It lit the heart of perceived reality. And in is e, piossed by means of minimal symb/'s a d numerals (like brushstrokes) applied $w$ ithomaximum discipline.

It's either 'right' or 'wrong'. Maths teaches you good discipline - to double-check. Unlike other subjects where there are grey areas, maths is straightion. An answer is either right or wron Dut this 'strict' nature of maths teaches you to be isciplined, to re-examine, to learn im. eediately from your mistakes. And remember, y. $:$ can always do better the next time. The best part of maths is that, on double-checking, you learn exactly where you went wrong.

Clear thinking shows the way. Like most other subjects, maths too must be learnt by paying attention and by concentrating. You cannot allow your mind to wander. In a literature class, if you have not been following closely, at the end you will wonder why Othello killed his wife. Maths too has its own plots. Only by understanding one part can you go on to the next. And by
understanding each part, you will be able to store it away in your memory.

Clear thinking at all times is a must. When exams approach if you panic, you immediately press a mental stop button that shuts off memo y and concentration. It's like a day when everyan 'ob seems to go wrong - if you panic, ygu ori't be able to do a thing. For example, if afire bre, ks out and you panic, it will blaze on. $B+b y$ leeping a cool mind, you will put out the firt $b$, throwing a heavy blanket on. Or in the is a fire extinguisher close at han you will seize it and use. Or you will sin ply all the fire brigade. Similarly, while swimm ng if you panic because a current is carrying y 1 away, you won't allow your brain to teliy that by swimming diagonally you can sare y urself. Or that you can try to attract the attehtion of a lifeguard. If he comes to your rescle and you hit out wildly or try to clins 'o nad, you won't be able to help him sa ve vou.


And let me tell you, even a genius cannot work if he or she is in a state of panic. If a problem seems tough, take a deep breath and address it with a cool mind-step by step and try to use your intuition. Even a hill is a mounta $n$ until you begin to climb it.

The bear went up the mounitain To see what he could see:
And what do you think he The other side of he molnt.in...

Let us march up the m unt in of maths. Once on it, we'll realise bow easy 1 is, how it broadens our horizon. And w 'll 'e able to see the other side of the mountain, which so far was in the shadows, out of our vision. It will enrich us individually anc the human race as a whole. There's ri e to pay here: just a recognition of the abilitiss locked within you that maths can open ithis shining key.

## 5

## Awakening the Mathe

 GeniusWe carry with us the wonders we seek wn'out us. SIR HO AS BKOWNE

Since thea cient Indians gave the world mathem tic , it is logical to presume that there was a $\&$ ret t aeal of mental activity in those times too ArH PS as a means of communicating this yst knownedge, it was woven into interesting to. es. Over the years, such tales came to be known collectively as Indian mythology. Unfortunately, by overemphasising the story aspect, much of the knowledge was lost as storytelling turned into pure art. Yet, you do find traces of it even today, as in the unabridged version of the Ramayana, which mentions an aeroplane or airship called Pushpak, and even goes into the details of aerodynamics.

Left side-Brain: Seat of your
numerical power
The power of ancient Indian thought lies in its recognition of a lifestyle that evolved from an amalgam of science and art. The brain ws perceived as the seat of power from $n_{1}$. emanated ideas, memory, reflection, plan ses, creativity, abstract forms ...

After decades of research, cciunti have finally zeroed in on this myste io s, jelly-lize, grey mass that resides in the sturi. Our great mathematician, Nature, ha. ble ssed all of us with two 'hemispheres' $O_{1}$ side in the brain. In a skilful weave, she the haped each side of the brain for specific tunstions.

The right st le or the brain guides the left side of the body while the left side of the brain guides the right ia of the body. Why this is so is not yet naessood.


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The right side can be loosely labelled as the 'creative side'. It is through this section that we perceive shapes, recognise faces, remember facts. The left side or the 'doer', allows us to talk, carry out linear tasks, and guides our day-to-dzy activities.

The two sides, however, are not exclasi e in their guiding process. They are inter onmed d, as neurosurgeon Dr Joseph Bogen caserihes, by a billion telephone lines.' Ancthe resfarcher, psychologist Robert Ornstein 1 as concluded that the left side guides us in our nonerical abilities, while the right side ern bles t to recognise designs.

## Developing spatial $s k$.'ls

The eternal op imist in me feels that it is never too late. Mathenatical learning should begin at home. in great advantage to be able to und ontw and reorganise two or threedin ens one sketches visually and follow the

space-time relationship factor. It's a skill that can be acquired by anyone.

Learning to read develops an important part of the brain. Along with reading, parents ought to introduce their children to shapes and size initially on flat surfaces, and later on a two three-dimensional level.

For example:
Step 1 :


Step 2 :


An anthronorvedist who visited an ancient tribe in Afrion t . ed sketching three-dimensional figures fo the tribes. But they were unable to comprel end them. The reason was that they had never 'eant to visualise an object by itself, sl atc ed on paper, as a three-dimensional figure. the part of the brain that needs to be trained had simply not been developed. It's been called spatial visualisation by scientists; simply put, it means being able to 'read' and restructure threedimensional figures. I call it the 'third eye'.

To get your child started on the road to spatial visualisation, buy toys which require 'engineering' skills. The break-and-rebuild type are excellent for this purpose. So are jigsaw puzzles. Girls shouldn't be limited to dolls and tea sets. If you want your child to develop the 'third eye', toys of all kinds are a must. You must also be prepared
for the complete dismemberment of the toy! It's no use nagging the child not to 'ruin' it. In the very act of dismantling and trying to put it back together, however unsuccessfully, lie the seeds of mathematics.

While taking apart a toy, the child had $t_{1}$ e opportunity to study its form. Buildins or rebuilding a set involves the brain in etting s.p a structure. Unfortunately, while brys are encouraged to play with su h nteresting little mind bogglers and brain bu luns, girls are banished to passive role $p$. ying with their dolls.

## Maths in sports

Sports activities zto a nother must for the budding mathematician. ollowing scores and working out averages he vecome an integral part of the sports sce, Whether it is tennis, football or crick- 1c,ares are constantly flashed on the telev sio. ineen and the scores or points, lengths mhps, speeds of sprints are also buttressed $w_{1}$,h comparisons of past records. This is an excellent and entertaining way to become familiar with fractions, percentages, and ratios. Today, the child has a variety of sports to choose and learn from - with tennis players serving at 225 kmph , bowlers delivering at 150 kmph and Formula I drivers hurtling at over 210 kmph . With glitzy charts displaying world records at a glance, there is no doubt that maths has come to stay in sports. I don't mean that the child will be able to apply, say, Carl Lewis' 9.86 -second sprint of 100 m in

1991, to a school maths assignment! But it certainly sends the numerals with their calculations spinning into the brain, thus laying the groundwork for interest and ability in maths.

Playing a sport is equally important. Settir a field to get the optimum result from the trajecto of the ball is a planned mathematical appronch. So is allowing the body to move, twist (or ev in pirouette) to catch a passing ball. Such rapid reflexes are not purely physi/ al thou they may appear to be. They need quic cye movements and intuitive calculation. th. $t$ are all directly related to maths conce ts. Such sports encompass awareness, imagery, co roination, intuition, all rolled into one flash if movement. And maths, after all, is about rajectories and velocities.

## The math anfabet

Get ar ald started on the road to developing the hiry \%e with symbol games which involve in le, flat or one-dimensional figures. This will eisable the child to feel comfortable and at home with symbols that are the alphabet of the maths language. To give a rough guideline, I have divided the exercise into three parts:

Symbol Analogy: Enables the child to find the numerical relationship between symbols.

Symbol Classification: Helps in classifying related symbols.

Symbol Serialisation: Guides the child in analysing the direction of a set of symbols.

## Symbol analogy

1. Question symbols:


From the answer symbols given below, as yo w child to choose the symbol that should be in p 'ace of the question mark.

Answer symbols:


Visual steps: By ex mining the numerical relationship between $t$,e first two symbols, you will see that it doubles. One circle becomes two circles. So, one s, are becomes two squares. The answer
2. Qu tim symbols:


Which one should come in place of the question mark? Choose from the Answer symbols.

Answer symbols:


Visual steps: The first symbol has five sides, the second has one side subtracted and has four sides. So the fourth box should have one side less than the third box. The answer is E .

## Symbol classification

1. Question symbols:


Which symbol is the odd man out?
Visual steps: The four lines A, B, C, be hing to one class. But the circle does not. So thearsuer is E .
2. Question symbols:


Which symbol is the odd man out?
Visual step: A, C, D, E are pointing horizontall up er down. But only one is pointing vertical'y down. So the odd man out is B.

## Sumbolsevilisation

Zuestion symbols:


Where should the dot be on E? Choose from the Answer symbols.

Answer symbols:


Visual steps: The dot moves in a clockwise direction. So the answer is $D$.
2. Question symbols:


From the Answer symbols, which one should in $E$ ?

Answer symbols:


Visual steps: Two $t$ ng are happening simultaneously in the quesion symbols: (i) the dots are increasing by ole, (ii) the square is changing direction. The answer is C since it fulfils both requirements.

By such ingy aious methods, you and your child will A $\quad$ op the third eye. Most of you have a 'Fami. Hour' when you play cards or word gan es. So why not include these 'symbol games' tou?

Later, you can move on to three-dimensional oymbol games, such as Turn the Box, which goes this way:


Which box from the Answer boxes is the same as the one above?

Answer boxes:


D
E
Visual steps: While you can reason out th answer, the right way to do it is to fix the in age of the first box in your mind. Now, mentaly rotate the image. When it clicks into pl ge, you will have found the right $\mathrm{b} x$, whych: E .

Please remember, the 0 ly simple mind games. If you or your hild, nd them too difficult, don't despair. It should n't stop you from taking higher maths. While 1 wo ald like you to develop your third eye tó its furlest potential, I wouldn't want you to be out off the subject because you experienced a bit of difficulty. Remember, even in the $m$ thematical world there are many fallarme. It -ould well be that 'spatial' visualisation is not an al-important factor in the learning of nac.s. Look at it this way - even the best writer in the world needn't be a perfect speller to love words, live words, and feel a sense of fulfilment as he writes.

That is exactly what I am doing. By introducing the world of symbols to you, I am making you and your child as comfortable with them as a writer is with words.

Take the word Om. Hindus believe it is the essence, the breath of the soul. The ' O ' is your lips shaping the word. The humming of the ' M ' is the vibrant undercurrent of life. Coincidentally perhaps, it is also the first letter of the wo d 'Maths'. Make maths a part of your life. An. all, it is the vibration that moves the entiren rid!

## 6

## Nurturing Mathabiliu

He who would learn to fly one dat, wist first learn to stand and walk and , wand camb and dance; one cannot fly mis flying. FRIEDRK'H NIE 7SCHE

Call it love at Mirrst sight or what you will, but when T w..s only three years old I fell in love with unib rs. It was as if nothing else existed as 1 vor,ked out sums. Everything else faded into b. vion as I got the right answers. I found a companionship that gave me a deep sense of emotional security and fulfilment. Perhaps I was too young to understand why this was so.

Now that I'm older and wiser, I can look back and understand what numbers meant to me. They were the rock that gave me strength. They were the numerous little soldiers that stood and defended me fearlessly. They were the pillars of my life. With numbers I knew I was safe, because no matter how the world changed, no matter what I heard or saw, I knew one unchanging force in my life would always be
there: numbers that gave me a beautiful message of hope - that two plus two always made four.

All of us need this constant factor in our lives. What bliss it is to feel that, come hell on high water, this factor never changes. It's the strong thread in our lives. It keeps us wholf wh $n$ other things could be shredding or falling to 0 des around us.

## The unbreakable thread

I want to share the mag on nombers with you because I have expe rience what they can do. Friends may come ana go, but numbers remain with you eternally. Th y are not just the symbols written in chalk on the blackboard. If you allow them to, thep will spring out and join you. They will guide vol into an adventurous world and thrill y $u$ with their variety and applications. They ohare a w/de range that can never be e co possed. They have a richness that you'd eyer dreamed of. Together with their many faces and phases, they have that one important quality that we look for in our friends, lovers, families - the constant factor.

Stability in an unstable world - that is what maths is all about. That is why I say maths is the pulse of our lives; the thread that keeps us together; the thread that never breaks.

Psychologists have harnessed this factor to bring solace to all the troubled minds that land on their couches. When a person is unhappy, he is told to make a list of what makes him sad.

He should also make a list of what he is blessed with. On this mathematical equation hangs the balance of pessimism and optimism.

I know a woman who has been paralysed for the past 20 years. Yet, she is always cheerfl 1 , always smiling, always interested in everytin. When I asked her how she kept her spisit up, she replied, 'Whenever I feel low, I rane a ist of all that I have to be thankful or That list includes loving, caring children et ough money to live on comfortably, a love'y home, and the greatest blessing of all - +6 b ciive to enjoy the beautiful world arou d m

Because she is pras sed, she is unable to do many of the things $t$, at others can. 'One day, I asked myself why I exist if I can't do so many things,' she cont rued. 'I thought for a long time. I know eare all here for a specific purpose. So wha wine? And the answer came to me: I wha a ere $t 5$ make others happy! And the only w. v. can make others happy is by being cheerful i. vself.'

Indeed, she has found the magic of a mathematical equation. It is:

Awareness + Cheerfulness $=$ Happiness
or

$$
A+C=H
$$

## Enumerating thoughts

Psychologists or psychiatrists ask us to make lists. In other words, they are asking us to enumerate our thoughts. By breaking down emotions into
simple arithmetic, we often find the solutions. That is why the woman I have spoken about makes a list of what she is blessed with. We make shopping lists so that we remember what we need to make our lives comfortable. We make a list of the pros and cons before taking important decision. For example, if you arso fered a new job, you write down certain prims.

| Old Job |  |
| :---: | :---: |
| Pros <br> 1. Known, trusted colleagues <br> 2. Confirmed job | L ower salary <br> 2. Sarne rut <br> 3. Family business, so no chance of accelerated promotion <br> 4. No travel <br> 5. No provident fund <br> 6. No perks |


|  | New Job |
| :--- | :--- |
| 1. Bros | Cons |
| 2. New challenge salary | 1. New faces |
| 3. Chances of faster | 2. Probation period |
| $\quad$ promotion |  |
| 4. Travel abroad |  |
| 5. Provident fund |  |
| 6. Perks |  |

Such a list instantly gives you the total picture. It also helps you to understand what is troubling you. Perhaps you can talk it over with your new boss, pointing out that with your wide experience, you needn't be put on probation.

Lists clean out the mind and make fore thinking possible. If a problem confronts you the best way to deal with it is to write aon n, breaking it into clear, demarcated se rments. The sheer philosophy and the ind ilt nat ire maths come to your aid, and offers vo ceiner a solution or the path to a solution

Why are such ists made? They have therapeutic value. De ision cannot be made when your mind isoclouded by emotions and confused thoughts. Math is tre ray of sunshịne that dispels those clouds and shows you the clear, blue sky that was lurnys there beneath the misty curtain in ypar ond.
derbapo that is why maths is seen as a cold, lir 'al, calculating science. But that's putting a quality to its face. It's like calling a surgeon cruel because he wields a scalpel on the operating table. Yet maths, like the surgeon, has a helpful, caring persona that shows you the light at the end of a long, dark tunnel.

When you are angry, you count under your breath: '1-2-3-4-5...' until you calm down. When you are unable to sleep, you count sheep. When you are depressed, you confide in your friend and feel better because, by sharing your worries, you have halved them.

What does it all mean? What does it all boil down to? That maths is the science of an art the art of living. Maths is inherent in your lifestyle. Maths is the obliging friend who shows you the way. It is only your emotions that have block d you from seeing what maths is. It is the coristr. it factor that brings stability to your life. It libe ates you from problems. It is a constant ompan, $n$ in your evolution.

## Equations of attitude

How does maths wotk for von? For a start, you must accept that it is alvays there for you to lean on - the cofist nt factor, the rock. Next, suppose you haw dream or a wish, how do you make itwo ne true? If you do not apply a positive mat'semitical equation to it, it won't come true. Here is how you do it in simple arithmetic:

Dream + Action $=$ Reality
D.aths tells you that the next step to a dream 1. +o act. The end result is the dream turning into ceality. Maths gives you the channel, the viaduct, the infrastructure to realise your dream.

You can further divide the first equation by setting goals to reach that reality.

Goal as in:
G: Go for it!
O: Over and above if required.
A: Another way if a dead end is reached.
L: Licked into shape!

In arithmetical terms, it would read as:
1
+1
+1
+1
$=4$$\quad$ or $1 \times 4=4$

What has maths done? It has be nith you at every step of the way. It has herp d 00 make a list. It has by its very equat. ni liberated your dreams. It has created as ${ }^{2} H_{0} s$ yucture for you to build upon. It has made vour way easier by further subdividing. It his belped you reach your dreamed-of destination l has made a reality of what you may inve thought was impossible.

When ypu h, ve been ridden with anxiety in the proceos $f$ veaching your goal, it has kept the picture n frant of you. It has held on to that goal en while you were awash with emotion. It ha. given you that gentle push when you m ered it because it refused to join those clouds in your mind. All along, it has moved you on steadily to the next goalpost. In short, it has always been there. And most importantly, it has never let you down.

## Maths within

The best part is that maths is not an outsider. It is within you, in your every heartbeat, in your brain. You have within you the maths-factor that has made you go on. It is the rock or the spine
that held steadfast while another part of you gave in.

Thosé who say, 'I'm poor at maths,' are doing themselves an injustice. They are only undermining themselves when they need not. It's like sayins, 'I'm poverty-stricken,' when a range of rich s glitters before you.

Those who say, 'Numbers scare m,' chould think again. Numbers have broughthros 10 where they are. They are the soldie ; f the borders of the mind, guarding it in timondiculties. They don't use guns or ammu itign, but something stronger and more lo g-lasung - faith.

Those who say, "I don't have the patience to do maths,' are rit ken. They view maths as an imposition an their time. It is not. It brings organisation infor your life. And it is because of this or malisation that you can go ahead perbats bigger things.
S. if you have a goal, don't let your emotions - le you and convince you otherwise. Miracles happen because you have worked towards them systematically. An artist shows you a finished, glowing canvas. He has worked on it with precision, with a structure. So should you. Hold on to the constant factor within you and allow it to liberate your dreams. Give yourself a chance by letting your maths-factor work for you. Make maths the pillar of your life!

## 7

## Language of Maths

Do you ralle that every time you say, 'I don't 'rave' a mathematical mind,' you are insulting your own intelligence? Let me explain owh
$M_{1}$ ths is a way of thinking. Its language is b. sed on numbers and symbols. Animals, our fourcoted friends, have a hazy concept of maths. Human beings, with their superior mental abilities, have developed this concept into practical science.

You may wonder how I can make the bold assumption that animals have some idea of maths. I will illustrate this with an incident I once witnessed.

A friend of mine has a beautiful Labrador bitch called Judy. One day, Judy delivered several adorable little pups. She licked each one and appeared to be very proud of her progeny. Since
there were too many pups for my friend to handle, she started giving them away. Judy was not around when her first pup was carried off by the new owner. Later, when she went to the basket for her daily loving lick, she paused, then she turne 1 to my friend and gave a little whine. You ou Judy knew instinctively that one pup was nis ing! Obviously she had never been to schosl on pe n taught maths. But she knew the d)fference between six pups and seven pups.

Judy's concept of maths w.o. guided by instinct. Human beings ha e 1 tob, but we have gone a few steps furth. Ouncophisticated brains have evolved it and sive. concrete shape to a way of thinking that is alrgady, within us. We have given it labels in the forn f numbers. Where Judy may realise a pupis nissing, we would say one pup is missing We vould also know that a pup here and a $p$ otbore make it two pups. Thus, we put it drw nuthematically as:

$$
1+1=2
$$



How easy it is for us today to say 'plus' or 'minus'! The two symbols ( + and - ) came into being in the fifteenth century. If a box of goods weighed more than it should, the merchants marked it as ' + '. This symbol was derived from the Latin word 'et', which means 'and', as i does in French.

When a box weighed less than it houto, tee merchant marked it as ' - '. It is posshle hat the minus sign derived from the ph is ign. True, minus could have been I or -, since 1 combination of the two gave + . Since $/$ cou d have been confused with the number one, the merchant settled for the horizontal -.

That brings me to the equal $\Leftrightarrow$ ) symbol. In 1557, mathemat cian Robert Recorde realised that two paralle lines of the same length were 'equal'. Thus, be vioce it down as $=$.

Conil back to Judy, her species has not bert able to evolve the maths-concept into a ie ce as we have. But the fact remains that she has a vague idea of maths. So, can you honestly say to yourself, 'I don't have a mathematical mind'? The simple truth is that each one of us possesses a mathematical mind. Today, even an illiterate person can say, 'I have three children.'

## The Maths Alphabet

Numbers and symbols are the alphabet of maths. The methods of adding, subtracting, multiplying, dividing are akin to word-building in a language. And the problems are like the sentences or
paragraphs of a language. Numbers and symbols may put you off maths, but consider this: a shorthand stenographer uses symbols to take dictation; a blind person, by feel, reads Braille through dot formations which, indeed, are symbols; even our language, or rather the art $f$ writing it, has symbols. We have:
. to denote a full stop.
, to denote a pause.
! to denote exclar at on.
? to denote a ratisn.
We read them ecery $12 y$, and they have become a part of ou thinking. If we didn't have these symbols, we w uldn't be able to write sentences. We vou 'In't be able to communicate. And maths is ommunication.

In $1 . g$ ge, we have writers who have evolva dorgs. Largely, our languages today are den ea from Latin or Sanskrit. Then we have the resional dialects.

Similarly in maths, we have mathematicians who have evolved the maths language, and the maths we learn today is a derivation or a progression. The only difference is that in maths we don't have dialects - except perhaps at a limited level. For example, as mentioned in Chapter 3, what in India is referred to as 'one lakh' is known as 'one hundred thousand' in the West. But these are mere labels. In the maths language, both a lakh and one hundred thousand would be written with the number one followed
by five zeros, with a slight variation in the placement of the commas:

$$
\begin{aligned}
& \text { India : } 1,00,000 \\
& \text { UK/US: } 100,000
\end{aligned}
$$

This is why a rich man in India is called 9 lakhpati. In the West, he is a 'millionaire'. 7 hese are labels derived from a single and easv in $d$. After all, it would be quite a moutn 1 to say: 'hundred thousandaire'!

In the English language, $\boldsymbol{u}$ i ave hau several great writers and poets, Ac's Charles Dickens, Jane Austen, William Shake pe re, to name a few, who clothed their iceas - based on their experiences - in bea titid words and sentences to convey their neaning to the reader.

Similarly, $m$ ths has its great luminaries such as Pascal Descartes, Eratosthenes. They too dressed their ideas, based on their experiences and e, nements, in numerals and problems to cs 10 y their message to the reader. But, before
g. On to quoting a passage from the works of one of these great masters, I'd like to give you a simple, comparative analysis between language and maths.

In a language, the alphabet is divided into vowels and consonants, as:

$$
\begin{aligned}
\text { Vowels }: & a, e, i, b, u \\
\text { Consonants: } & b, c, d, f, g, h, \text { and } \\
& \text { on through to } z
\end{aligned}
$$

Similarly in maths, you have numbers. They are divided into prime numbers and composite numbers, as:

Prime numbers $: 2,3,5,7,11,13$
Composite numbers: 9, 15, 25, 32, and so on

As you will see, prime numbers are an entity in themselves. They cannot be divided. You ca divide $2,3,5,7,11$ and 13 by any nyner.

Composite numbers are divisible. For exam ${ }_{f}$ le:

$$
\begin{aligned}
\frac{9}{3} & =3 \\
\frac{15}{3} & =3 \text { on } \frac{15}{5}=3 \\
\frac{25}{5} & =3 \\
\frac{2}{2} & =16, \text { and so on. }
\end{aligned}
$$

In setion ve are taught this with words such as 'ipteg-s' 'actors', etc. I have deliberately kept it $s \mathrm{mp})^{+}$make for better understanding.

## E atosthenes' sieve

Now that you have grasped the 'vowels' and 'consonants' of maths, you will enjoy the passage I am about to quote from one of the great masters of mathematics.

About twenty-two centuries ago, there was a Greek geographer and astronomer called Eratosthenes. He invented a sieve to sift and filter the prime numbers from the composite numbers. It was a beautiful piece of art. He wrote the composite numbers in six columns. He circled the
primes, crossing out all multiples of 2 by a clever stroke of his pen. Similarly, he circled the number 3 and crossed out all multiples of 3 , and so also the multiples of 5 and 7. Finally, the remaining circled numbers were the primes.

Just as Dickens formulated sentences A the English language, Eratosthenes formula ed mathematical sentences. Dickens dazole 1 to with his evocative prose, Eratosthenes dio ze a with his evocative mathematical $\mathrm{p} \cdot 0$.

I do agree that you ray $\mathrm{f}_{1}$, d that reading Eratosthenes' passage is no the same as reading a passage by Dicken. I ere I'd appeal to you to call upon that aesthetic sense you are blessed with and use it the way you would to 'read' or view a Picasso printirt You'd admire the artist's clean brush strkic o can you admire the fine, clean lines of Fir osthenes' sieve. I am sure a maths clite ar cit $c^{\prime}$ would write about it this way:
in a revolutionary, trailblazing method, the geographer and astronomer, Eratosthenes, has tapped the essence of mathematics. He has captured the inherent dynamics of maths with his pen. The clarity with which it has been set down will motivate maths fans to venture into areas never before explored.
With this unique sieve, Eratosthenes lays down a clear, shining, positive approach. With one glance, maths aficionados can now find practical, creative solutions to
problems. By inventing the wheel of mathematics, Eratosthenes has abolished the need to reinvent it at every stage for every problem.
Now read Eratosthenes' sieve:


If you have gone through the sieve and digested the tiniest morsel from it, it means that you have understood the spirit of this chapter. Earlier, you might have looked at Eratosthenes' sieve simply as a box of numerals with arrows crisscrossing it, just flipped the page over or even
closed the book. If you had, remember, it would not have been because you don't possess a mathematical brain but because that part of your brain was not stimulated enough, not for the lack of ability, but for the lack of interest. It is not that you lacked potential, but that you were li below your potential.

## The fourth dimension

My idea is to show you tha, y have within you a greater energy, a ghat-r retivity, a greater problem-solving capa ity th an you give yourself credit for. You have th third eye but have not used it so far. You ha, vewed the world threedimensionally. As you read this book, I want you to feel the four dimension within you.

You ma, wonder how opening your third eye would rive you a fourth dimension? Sounds llorgca. cueen't it? But you will understand what IA er when I tell you that the two eyes you p issess show you a three-dimensional world. This point was brought home when a relative had an accident. He had to have one eye bandaged. He could see clearly with his good eye, but when I unthinkingly held out a photograph which I wanted him to see, in the act of taking it from me, he missed it by a few inches.

Similarly, I feel that by keeping the third eye closed, you miss grasping maths by a few inches. All you have to do is open the third eye!

## 8

## Mastering Mathabilitu

They can because they think ry ar. VIRGIL

Ihave sai ovy and over again, in different ways ha ach one possesses a mathematical brain We brye the seeds of maths, the ingrained poyder fy inderstanding the subject, deep within us low another question arises. How do we cral out that power so that we can drive away that feeling of alienness, and use it to its fullest potential? How do we turn ourselves from maths aliens into maths lovers?

I always had the answers in my mind but did not have the right words to express them. One day, however, while reading the newspaper, my eye fell on a beautiful line in an advertisement. It read: 'Magic of drip irrigation - turning deserts into goldmines'. In a flash, I had found the words! I knew now how I should convey those answers! Paragraphs formed in my mind, words jostled and collided with thoughts.

## Mental Drip Irrigation: The four Cs

If I liken that part of the brain that lies unused due to maths alienness to a desert, then the cure lies in mental drip irrigation! I want your brain to soak and absorb the water of knowled 1 will give it to you, drop by drop.

To irrigate the desert and turn it into ... bla hs goldmine, you need only four drops of warer. Let each one drip into you, and your thi sty brain will open its folds to drink what are those four drops of water?

They are the fou. Cs:
Common sense
Confidence
Concerira ion

## Con'rl

Su pri,ed? But that is all you need to turn y un el! into a mathematician! I will take each 1ro? on my finger so that you can see it shammering and illuminating your inner mind, thus opening your third eye.

Common sense: Every maths problem needs a good strong dose of this special drop. It tells you that nothing is beyond your ken. If a problem is set before you, read it carefully. Remember, it need not freeze your mind into a state of panic. It is a problem that has a solution. All you have to do is apply your common sense to it.

To help you draw on this drop, let me ask you a mathematical question:

It 6 men can pack 6 packets of candy in 6 minutes, how many men are required to pack 60 packets in 60 minutes?

Read it carefully. Now apply your commont sense. The first thing that should strike you 's that you do not necessarily need more men. Nh )? Because you have more time to pack thetrat iy!

Now write it step by step:

1. 6 men pack 6 boxes in m nute;
2. 6 men pack 1 box in 1 mh ute
3. 6 men pack 60 boxes in 60 minutes.

Painless, wasn' $1 ?$ To the maths-alien, however, it appears to e a complex problem. She reads it, feels that is too difficult and clams up mentally als. the robotic manner of teaching maths in c. 00 doesn't help, and the clever way in which the problem has been worded makes gou fou is on the number of men immediately. But a $\mathrm{tt}_{\mathrm{t}}$ thinking irrigated by common sense, will s.on you that this is not so.

If it had been worded in another way, you would have found it simpler:

If 6 men can pack 6 packets of candy in 6 minutes, how many packets of candy would they pack in 60 minutes?

Knowing that the number of men remains constant, you'd write:

1. In 6 minutes, 6 packets are packed.
2. In 1 minute, 1 packet is packed.
3. In 60 minutes, 60 packets are packed.

But the way it was worded the first time calls for that one drop of common sense! So you see, it is as easy as eating candy!

Confidence. Another drop required to tackle a maths problem is confidence. The great confidence of a bulldozer. Plough into a prosic with the firm, strong feeling in you that you can do it. Nobody, not even yourself, wih uare y question you!

I notice that many people pel the e s 'trick' to solving a maths problem. Whe someone shows you how to solve a seemi sly complex problem, the human ego come into play. She makes it appear so easy, you an to crawl into a corner to hide yourself. You vonder why she found it so easy, and jou lidn't? What your 'tricky' problem-solver $h_{\text {, }}, \rho$ conveniently forgotten to tell you is thet ho had come across it earlier than you and bat taken the time to get a grasp of fi, At at she may have stumbled on to it by ace de at. She leaves out these tiny details and makes you feel like a 'dumbo'.

While the 'tricky' problem-solver may appear to be handing you a method on a silver platter, be wary. In the long run, her method may not work. It will not work if you have merely adapted it but not understood it. It will not work even for her, because she sees it as a clever trick or short cut. Which means that for each problem she has to find a new 'trick'.

The best way to learn maths is to allow your natural intelligence to take over. With a clear, confident mind, you will be able to do it.

Concentration. Every Indian, when confronted with the drop of concentration, will instantly think of Prince Arjuna in the Mahabharata. It is an oftrepeated tale, but I must narrate it here to underline the importance of concentration.

Dronacharya taught archery to the ro, as princes of Hastinapura. One day, he decia. do test their power of concentration. G2 hering wis royal pupils around him, he pointed to the target - a bird on the branch of a tee. Trev had to shoot the bird and bring it aven. The first candidate for the concer ati n test was Prince Yudhisthira.
'What do you set's a ked Drona, as the prince readied himself with the bow and arrow.
'I see the ird, the branch on which it sits, and the leaves of the tree,' replied the prince.

'What else?' demanded Drona.
'The tree, the sky, you - my teacher - and my brothers,' said Yudhisthira.

Drona called the other princes - Duryodhana and Bhima. They gave the same reply. Poor Dror a was saddened. He felt he had failed as a teachur.

Finally, with a heavy heart, he called pr ice Arjuna.
'Do you see the bird?' he asinet bopelessly. 'I do,' replied the prince, confidently. 'What else do you Su??' shed Drona.
'I see only the bu d,' repried Prince Arjuna steadfastly.
'Do you oot/see the tree, the sky, myself your teacher - and beyond?' queried Drona, hope lifting his keart.
'I see Only the bird,' repeated Arjuna stubormis

What part of the bird do you see?'
Only the eye!' said Arjuna.
This was the answer Drona had hoped for. It displayed Prince Arjuna's power of concentration.

What does concentration do? It helps you to focus. It blocks out anxiety and emotions. If a sportsman were to worry about being tackled by his opponents as he runs for the goal, he would never get started. Think of maths as a fun-sport. Your goal is to get at the solution, to net it.

The point I am making is that when a maths problem is presented to a person, she allows her
anxiety to take over. She creates those exceptional circumstances in her mind. Instead, if the emotions were blocked out by concentrating, he or she would set out calmly to understand the nature of the maths problem and work on a method o solve it.

The best way to concentrate is to swito 20 neutral gear. Don't think in terms of "On God, how will I do it?' or 'What if I ca no ${ }^{+}$do it?' Be aware that though you are a ligh y intelligent person, it is not your intelligen e mat is on trial. Read each sentence wit concentration. For example, if the prob $m$ is ritten thus:

A woman spend is 230.50 on an average during the fitit 8 months. During the next 4 months sh spends Rs 180 on an average. During hat year, she takes a loan of Rs 164. What the average monthly income of that
F. ost, read the problem carefully. Next, e. unerate data. Then allow your common sense and confidence to tell you that you can do it.

Concentrate on the problem, and begin by outlining the data in simple steps. Instantly, you have the answer:

1. Sum of money she spends in 8 months

$$
8 \times 230.50=1844
$$

2. Sum of money she spends in the next 4 months

$$
4 \times 180=720
$$

3. Total money spent in 12 months

$$
1844+720=2564
$$

4. Loan taken is not part of her income. Therefore,

$$
2564-164=2400
$$

5. Thus her monthly income

$$
\frac{2400}{12}=\operatorname{Rs} 200
$$

Why is concentration so import $\mathrm{nl}^{\circ}$ It acts as a catalyst towards defusing the 'ran't-do-it feeling. Negative thoughts ma.e you focus on the irrelevant. Instead, if vou tring your powers of concentration to bear on the maths problem, you shift your focus to what is relevant.

Sometimes, wo ls and numbers defeat you at the first readrg. But if you don't allow such defeatist h ughts to creep in, you are already on the igt path. Confidence in yourself will drie $y, n$ to focus on the right clue. It will enable vor th see the problem in simple terms. This keeps $t_{1}$. feeling of frustration and the sense of defeat at bay.

Decimals might be another reason for you to feel discouraged. You feel that if you were given round figures, you would be able to solve the maths problem easily. Fine. Think along those lines.

Suppose the woman spends Rs 200 (instead of 230.50 ) in the first 8 months, how would you tackle it? You would multiply 200 by 8 . Write that down. Now apply the same method to
the actual problem. So you will write it now as $230.50 \times 8$.

Simplifying a problem makes you understand it better. With understanding comes the feeling that it is manageable. You can simplify it if you concentrate on it.

Control. The first three Cs - Commons inse, Confidence and Concentration - add up to the fourth C - Control. Once you the doused anxiety, fear, panic with th ose the ee recious drops, you will automatically fer in control of the situation. But you mus feet you are in control. You must tell youses that you are in control.

Look on maths a a purring sports car. You are the driver. You are at the helm. There's no way you are oing to relinquish the wheel to another. If rou are driving uphill, you will drive slowly. Sminarly in maths, take your time. Unden tap each number and symbol. Analyse wha is required. You, the controller, have three trigs attached to your fingertips - the data, the method, the solution. As you take each step, or go into the next gear, you will find it gets easier.

How do you always ensure that you are in control? If you find that despite taking the step-by-step approach, you have reached a dead end, don't give up. Go to an easier problem. An easy problem gives you that sense of control because you know you can solve it. Often, a shift in gears will give you a fresh perspective on how to solve the more complex problem.

I know a writer of short stories who decided to try her hand at writing a novel. After a few months of intense scribbling, she found she couldn't go on. As she told me, 'I felt I had lost control of the characters, the plot. I didn't know in which direction my story was heading.' Wha did she do? Did she give up writing altog ther? Not at all. She put the novel aside and wantb. ck to writing short stories. She regained co triol. She then approached the novel with in sb ne sense of being at the helm. She took $u_{1}, t$ boon, chapter by chapter, and within sin mons, she had finished her book!

The sense of contro is emportant not only in your approach to math but to everything in life. If you let the founds, the four life-giving drops irrigate you min $f$, you will never regret it. Your mind will loom with the knowledge given by these drops. And you will find that it is not only enat's cha you wish to explore, but other areas as ve $1!$ After all, life is an eternal voyage of selfa scovery.

## 9

## Mathability: Problem

## Solving

It isn't that they cma't ore solution, it's that they $\mathrm{can}^{\prime} \mathrm{t}$ oe the problem.

> G.K. C. ESTERTON

Each one of $u$ is blessed with that intangible ability c. N1ed intuition. It is a mother's intuition that tel hen why the baby is crying. When your can su larn'y starts making strange knocking so in. 's, it is the mechanic's intuition that tells 1 im . what has come loose. When there is a power failure, it is the electrician's intuition that tells him what could be wrong and where he should start.

Solving maths problems too requires intuition. My statement may come as a surprise to you. All this time you thought that maths was a matter of memorising formulae, using a logical, step-bystep approach, and that was it.

## Mathematical mind games

But think about it. What exactly is intuition? What is that special flash that illuminates a problem
and shows you the way? Is it a gift from God? It is - like everything else. Intuition is seeing relationships based on personal experiences.

For example, how would you solve this problem:

If 5 tyres were used on a car which has travellec. 20,000 miles, how many miles would each yo sustain, if all the tyres were used e ua ly in sustaining this mileage?

If you approach this problom, nurely 'logically' without seeing relationshi ${ }^{5}$ s, you would simply divide 20,000 by 5 and ome up with the answer: 4000 miles. How did vo yarrive at this answer? By a simple formula of division. But you got the wrong answer.

Here is there your experience and the ability to work out relatronships come in handy. For example, you mon wonder if the car in this problem runs of the tyses instead of the normal four!

The intuitive person, however, will work it out in this way, to arrive at the right answer:

- When the car travels one mile, each of the 4 tyres sustains one mile's use.
- So when a car travels 20,000 miles, a total of $(20,000 \times 4) 80,000$ tyre miles are used.
- Since the mileage has been gathered on 5 tyres, each tyre has used

$$
\frac{80,000}{5}=16,000 \text { miles. }
$$



Another intuitive reison may solve the problem this, way:

Since 4 ou of 5 tyres are being used:
$20,000=16,000$ miles.
What and you deduce from this exercise? If you 'uswer was 4000 miles, don't be tis eartened. At the first reading, you were pobably relieved that it was a simple matter of division. Without stopping to think, you rushed into that formula, thinking: 'Ah, for once, I've got it!' Had you stopped to analyse it, I am sure you would have realised it was wrong.

Your instant response is the human factor I have talked about all along. The image of maths that is so prevalent, is that it is a mechanical process.

## Using your imagination

If we had been taught from an early age that maths is a living, vibrant science, I am sure we would have seen it in its totality. And in viewing it as a larger practical science, we would have collectively developed the maths intuition

As it stands, in class or at home, child n are made to do maths in an automatic, u in agiative way. With the idea fixed in their anc that 'the teacher's method is the righ. o 'e, st wients do not develop their own thing Or even realise that it is possible to do su Vtry rarely are they encouraged to work ou their own methods. Thus, while they add, syb ac multiply, divide, they do it mechavica ${ }^{17} \mathrm{v}$, like a calculator rather than as thinking po sons.

Since tr is happens to many people, those who display $m$ re ninking powers, more imagination, morf intition, are seen as having a 'mathematical min '. NC $t$ appears to be a complex problem to Sheila is child's play to Rajiv. The fact is that $t_{1}$ 2 quality of exposure to maths differs between these two people. As a result, Rajiv makes connections. He sees the links or relationships in problems that Sheila does not.

What adults fail to teach children is that a formula is purely a means to solving a problem. It is not the be-all and end-all of maths. For example, Arun, a geology student, tells me about a new problem he was given in school:

If 2 men can build a 10 -feet high wall in 20 days, how long will 5 men take to build the same wall?

Arun says: 'I remember this vividly because I was the only student to get the right answer. Even the ones we saw as the "brains" of maths rushed to answer, 50 days! At home, we had some construction work going on. I remember ny father telling the contractor to put more mer the job to get the work done faster. So $W$. rked it out the other way and figured the answe los 8 days.'

As you can see, Arun's per son، 1 experience came in handy in maths He w s.o.ble to link the fact that more men woun ta ce less time to do the same job. In this omme sense factor lie the seeds of what is moth matically called inverse proportion. Unfortunat 1 y, students are not always taught a formula rin a common-sense approach. The result: Hey memorise the formula and are then unzele to use it or apply it when different probler c arise.

## C. nc ptualising problems

In cuition is all about understanding through experience. Haven't you often said: 'If I had known this in school, I would have understood it better?' Now that you have experienced that something, you are able to apply it to the theory you were taught in school. What you are now experiencing is that special flash of insight which has risen from your intuition working on the wheels of your experience.

In the practice of medicine, an experienced doctor intuitively knows what could be wrong with a patient. Based on this and his examination,
he teils the patient to carry out specific tests. A less experienced doctor might not get this flash of intuition and may have to ask for a wider range of tests to be carried out. Similarly, a 'mathsdoctor' practises maths based on his own experience.

This is interesting, because it means you are never too old to learn maths. Unlike a stude th, an adult's approach to maths is exp rience. And that is certainly a plus point! Io tact, I see life as an eternal learning process. Re dless of age, we are all students. Peop, wi o say, 'I'm too old to learn,' are only a mean ing their maturity.

An adult knows ha, his fixed deposit amount in the bank yields s.f, 11 per cent. Here, he experiences for him elf the theory of percentages that he haplea in in school. But in school, he had abseto the method of percentages purely as an abshest mathematical formula. As I have said earliv, intuition arises out of the quality of e no yre to maths.

Start with simple problems. Then go on to more complex ones. Collect bits and pieces of information and punch them into your memory. Then, whenever you come across a problem, you can sort through your 'knowledge-bank', using your experience and common sense creatively to solve it.

The right solution is there in the brain, but maths-aliens do not give themselves enough time to think and thus perceive the solution.

The very act of perceiving an idea, the thinking process, is called conceptualisation. It is the vital
key to problem-solving. It is a mental lever. You weigh one idea. If it does not work, you try another. The greater the choice of concepts that you give yourself, the greater the chances of your solving a problem.

## Overcoming mental blocks

Quite often, however, a person comes $p$ against a mental or conceptual block. Tha teacon could be panic, a negative appr ash, whe read incorrectly, or the wrong fou. rroblem-setters use this human factor anc exploit it to lead you into what appeass is be a daunting maze. Actually, the proble n-s tter is giving you an opportunityo to use your intelligence, your analytical pow rs.

To solve a pioblem you may have to use a familiar p.oceos in a new way, you may have to abar in a conventional idea, you may even have to reass,nile certain components to make a new irn ngement . . . . The rugby player's approach dues not work, though it looks tempting enough. it brings you back to square one. Whereas, if you think for five minutes longer and work at it systematically, chances are you will be closer to reaching your goal in less time.

If you feel you still have a mental block, form an image in your mind. Suppose you were confronted with a high wall. What would you do? Would you beat your head against it? No. Would you walk away from it? No. You would try to find a way to walk around it or climb over it.

Similarly, in maths, try to walk around the mental block. By opening your mind to a new approach, you will be engaging in a creative process. You will be taking a subconscious leap. A leap, not a mere step. It is a geometrical, not an arithmetical progression. You have taken ins leap after a good deal of thought, after in uch stirring in your brain, after a host of deas ha ve whirled, shifted, collided, and finally een ged as a pattern. That is when you en er enc that special flash - intuition!

## The hungry dogs!

Let me animate it for you. You have two dogs called Rambo ain Hammer. You have tied both of them on on short leash.


The diagram above shows that Rambo and Hammer are at the centre of the line C. On either side you have placed a large bowl of dog food - $A$ is one bowl and $B$ is the other. The two dogs are hungry. What do they do? Rambo tries to reach A, while Hammer tries to reach B. Thus:

## Diagram 2



Neither can reach its respective bown. The leash is too short. They try again, batuey:

Diagram 3


No luck. ${ }^{\circ}$ Har ory, they sit down at C once again, and bo, at each other nonplussed:

Diarram 4


They have to think of a new way to appease their hunger. Finally, both turn to bowl A and eat:

Diagram 5


Then both turn to the bowl $B$ to eat:

Diagram 6


By trying, thinking, analysing, Rambo and Hammer had a flash of intuition which th. $y$ carried out in Diagrams 5 and 61

So you see, intuition is enceldeo ir ach one of us. Sometimes, or mon oft han not, it arises from a practical situation a d collides with maths which is seen as af at trast science. Knowledge and intuition are inte. $w$ hed in a constant cyclic motion. Whal yo should do is hook on to it and the world of $m$ ths will open out to you in one big, gloripis, grand flash of intuition!
You can do it!

## 10

# Logic of Mathabilites 

## Nothing is particularly hayn (1v AI divide it into sma' obs.

Do you Rnowsomething? I envy you. You are on tran e of a discovery. Looking at maths again $w+b$ fosh eyes is rather like going out on You fin + date. It has all the breathless excitement th. ${ }^{+}$bees with it. The introductions have been made. The spark has been lit. Now, it's a progression.

The best part about relearning maths is its uncanny resemblance to a first date, where you hang on to each other's words. You talk about your likes and dislikes. You discover the things you have in common. In other words, you talk about yourself as you are. Then, an interested query from your date leads you back to your childhood, and back again to the present. Perhaps you touch on the future.

That is what learning or relearning maths in adulthood is all about. You bring to it your experience, and start somewhere in the 'middle', with a feeling of exhilaration because you don't have to start again, from the beginnins Somewhere along the way, you call on what yo have learnt in the past. And you realise mat the grounding makes sense to you now afto all the, e years!

Most people sail through a dd ng, subracting, multiplying, and dividin, The take to fractions like a fish to water. They dive into the sea of numbers, flick their hins and pick out simple fractions. It is worde ful to know, for example, that half of a pie written as $\frac{1}{2}$. And that it denominates one part of an entire piece. The numberatove the line is the 'part' and is called the ramer20; while the number below the line is ti 'ol. 1 e' and is called the denominator.

## Mustering the methods

The beauty of the language of maths lies in its short, precise terms. If you were to put it into words, you would be writing long essays that would only confuse you! In its essence, the maths language saves you both time and energy. As you master it and its methods you can, within minutes, arrive at your answer.

Every fraction has a wealth of meaning to it. For example, $\frac{1}{2}$ can mean:
a. Sharing:

One half of a whole pie.

b. Computing:

One amongst others. For examply, you ean say 'one out of two childan 0 every day has brown eyes.'

c. Comparisern

One con ared to another (ratio).
d. Moth malical:

One divided by two.


$$
1 \div 2
$$

It is all so simple. Where is the barrier? After talking to several people, I discovered that the confusion begins when they first learn fractions, decimals, and percentages.

The problem in understanding, as I see it, arises when a child is taught addition, subtraction, multiplication, and division as independent segments. So the mind is set on that course. Every
new segment that is taught thereafter is seen as independent in itself.

Yet, fractions, decimals, and percentages converge, unlike the first four segments. For example, you cannot change adding into subtracting. Yet, you are asked to change frac 101 s into decimals or percentages. It looks dilfic $\cdot \frac{1}{2}$, because in your mind, fractions, definals. at. d percentages should stand apart from une apother.

What many fail to understa d is that cactions, decimals, and percentage a $t_{1}$ o same segment, but expressed in a diffe ent way. They are translations of each othe not rransformations. They are another way of an er canding and expressing the same thing

In the En- lis ' language, you can express the word possibs as) conceivable or probable, depending on the context. Similarly, in maths, fractions, decirna's percentages are used, depending on the context. The differences are slight, but 1. p p rtant.

In decimals, the denominator is always in tens. For example, you can write a fraction as $\frac{1}{2}, \frac{1}{4}, \frac{7}{6}$, You cannot do this with decimals. Here, tens, hundreds, thousands, etc., are always the whole or the denominator. For example:

Decimal Fraction

$$
\begin{aligned}
0.1 & =\frac{1}{10} \\
0.01 & =\frac{1}{100}
\end{aligned}
$$

$$
0.001=\frac{1}{1000} .
$$

Percentage is a specialised department of fractions and decimals. Here, the denominator always 100:

| Percentage | Fraction | Decimal |
| :---: | :---: | :---: |
| $4 \%$ | $\frac{4}{100}$ or $\frac{1}{25}$ |  |

## The scaffolding element

So far, so good. But what , nathematician like me takes for granted is otas simple for a person whose mathematica' brain has not been stimulated. This curyed on me when Sheetal, a born-again math atician, explained to me that in school s. e bad been taught to divide fractions by in erting the denominator and then mul ip. ing it with the numerator.

What Sheetal meant was that if she were gyen a problem thus:

Problem: $\frac{5}{4} \div \frac{4}{5}=$ ?
She was told the method was:
Method: $\quad \frac{5}{4} \div \frac{4}{5}=\frac{5}{4} \times \frac{5}{4}=\frac{25}{16}$
She memorised the method but never understood it. Sheetal has an enquiring mind. Recently she decided to work it out for herself, and this is how she did it:

Problem:

$$
\begin{gathered}
\frac{5}{4} \div \int \frac{4}{5}=? \\
\text { (numerate) } \div(\text { denominator) }
\end{gathered}
$$

Then she f:gu ed that the denominator $\frac{4}{5}$ had to begrar od to $\frac{5}{4}$. She saw it as a number in tse and wrote it as:

$$
\frac{4}{5} \times \frac{5}{4}=\frac{20}{20}=1 .
$$

Dimly, she began to perceive the importance of this. So she wrote it down, inverting both fractions:

$$
\begin{aligned}
\frac{5}{4} \div \frac{4}{5} & =\left(\frac{5}{4} \times \frac{5}{4}\right) \div\left(\frac{4}{5} \times \frac{5}{4}\right) \\
& =\frac{25}{16} \div \frac{20}{20} \\
& =\frac{25}{16} \div 1=\frac{25}{16}
\end{aligned}
$$

'Wasn't that rather a long process to get the same answer?' I asked.
'Of course it was,' she replied. 'But that is where I felt my maths teacher had turned us into robots. He never explained why we had to inve t the denominator. But after working out seren fractions using my method, I realised that hat he had taught me was correct.'

I must have looked a little a word. She explained, 'Don't you see? M, m ths teacher was handing me a short cut. His inte trons were good. Why teach the children the gowinded method when he could make ife eavier for us? He was handing us a finished pr duct without explaining what went into the $n$ aking of it.'

Suddenly sam what she meant. But she insisted on giving ne an example: 'I see all students as little "rachers engineers". The teacher gives us a short +2 a bridge. But the next time we have lo bila aridge on a different level, we will not be bly - to do it because we don't have the support sy tem or the scaffolding to build it on. We don't inderstand it, so we cannot do it.'

## Einstein and the little girl

I think you have understood what Sheetal meant. In fact, I have often wondered since, how much of maths we have to unlearn to learn it again! I think this point was understood by Albert Einstein. The city he lived in - Princeton abounds with stories of this great man. One such tale is particularly striking, and I think you will enjoy it.

A little girl would often visit Einstein. Her mother, on realising what her daughter was doing, questioned her about it.

The little girl replied, 'I had trouble with my arithmetic homework. Someone said that at No. 112 lives a very big mathematician and that is also a good man.' The mother listened vith dawning horror as her little daughter continu d artlessly, 'So I went to him and asked hion to help me. He explained everything $v, r$ rell. I understood it much better than wor our teacher explained it to us in scrion. $\mathrm{H}_{3}$, said I should come to him whenew I f.wnd a problem too difficult.'

Upset at what she aw as a presumption on her daughter's mart, the agitated mother rushed to the hallowed $1>0.112$ and apologised profusely for her Auchter's behaviour.


Einstein said gently, 'You don't have to excuse yourself. I have certainly learnt more from the conversation with the child, than she did from me.'

I have often wondered if the great man realised the value of the 'scaffolding element trit Sheetal explained to me.

## Avoid reinventing the wheel

Yet, I do realise that in maths er ain things have to be taken as 'givens'. ha/sndy at had to spend time questioning and sarnis yoout every 'given,' he would never progres boyond a certain point. He might understard. few formulae thoroughly, but he wouldn't have more knowledge, only a limited storgor it,

Thers\% feel that you should approach maths con your individual point of view. For exa np. Sheetal's way of working out the or br ms may seem to you like reinventing the u heel. You know that to divide a fraction by a fraction, you have to invert the denominator and multiply the numerator with it thus:

$$
\frac{2}{5} \div \frac{4}{3}=\frac{2}{5} \times \frac{3}{4}=\frac{6}{20}
$$

It has not given you any trouble so far. It has become a part of you, so you may rightly feel that to follow Sheetal's method at this stage would be rather confusing. As I said, tackle problems that you haven't understood. Sheetal has her scaffoldings; you may require your own
in other areas. Choose the ones that puzzle you and go on from there.

I feel that the 'missing link' between understanding and solving a problem often lies in one thing alone: the student has not understoo ${ }^{7}$ the actual meaning of words like fraction. decimals and percentages. This is the reason by I started this chapter by defining each term and giving 'translated' examples.

It is just like learning to 1 ri e $a$ c.... Maths has different gears. When your verse a car you go into reverse gear. The ability to 'reverse the mental gear' sometime. acts as a brake on the student. Maths give problems that need a different gear or different approach each time.

Let me mus rate this with a problem:

## Problem

In art a nination, $75 \%$ of the candidates rass dir English, $65 \%$ in Mathematics, while

1. \% failed in both English and Maths. If 495 students passed in both subjects, find the total number of candidates who took the exam.

To the maths-alien, this problem appears complex. But remember, percentages are there to simplify the matter. You will see what I mean as I give the step-by-step approach.

Method:
Suppose the number of candidates $=100$, 75 passed in English. So, 25 failed in it.
65 passed in Maths. So, 35 failed in it.
15 failed in English and Maths.

Candidates failing in English alone $=$

$$
25-15=10
$$

Candidates failing in Maths alone $=$

$$
35-15=20
$$

So, number of failed candidates $=$

$$
10+20+15=45
$$

No. of passed candidates $=$

$$
100-45=55
$$

If 55 passed from the total of 495 passed from the total is.

$$
\frac{100 \times 495}{55}=50 .
$$

So, the total number tho gat for the exam $=900$
Percentage or a hu dred, gives you a base or a yardstick tor lculate the rest. Thus, $75 \%$ means 75 very hundred. The last step may appear complex. But if you write it in words, it goe.

55 passed of 100
495 passed of X.
Now, assemble the 'passes' together. Next, put the totals together:

$$
55 \text { and } 495 ; 100 \text { and } X
$$

Then you put the ratio signs to make it clear.

$$
55: 495:: 100: X
$$

This reads as, 55 is to 495 is the same as 100 is to $X$.
Translated:

| Symbol | Meaning |
| :--- | :--- |
| $:$ | is to |
| $::$ | is the same as |

Once you have understood the relationship between the 'pass' and the 'total', the next step is easy. If 495 is multiplied by 100 , it gives 49500 which cannot be the answer. When divided by 55 , it gives the exact relationship to 49500 .

Thus:

$$
\frac{49500}{55}=900
$$

As Sheetal puts it: 'maths is like $\mathrm{H}_{\mathrm{C}}$ u . ng out how you are related to such or 'sych cousin and so on!'
Now do you see rhat I meant when I said relearning maths is ry nuch like setting out on your first a te. You have certain assumptions y you mind. You link them up elegantly. bore you know it, you are looking forwar 1 to ysur second date!

## 11

## Mathability: Making

## it Easier

Turn your stumu'irs hocks into stepring sines.

ANO ymOUS

over the yeary, I have talked to various people abou . hs. I find that most people face no proulery with addition, subtraction and mu. ipl yalion. But for some reason, division creates a. light setback. I have often wondered about $t_{1}$ is. One reason could be that students feel the different methods are taught in the order of difficulty. Addition being the easiest is taught first, followed by the slightly more difficult method subtraction. Next is the more complex method multiplication, and finally, the most difficult method-division.

This is not true. The sequence of teaching has nothing to do with which method is easier or more difficult. It is a natural progression. For example:

Subtraction is the reverse of addition.
Multiplication is a progression of addition.
In addition, you write:

$$
2+2+2+2=8
$$

In multiplication, it is:

$$
2 \times 4=8
$$

Here, the 4 denotes 4 times. Hence, he tables we memorise are actually quick montar a gitions.

Division is the reverse on $m$ ultipliation.
For example, in muti $y \omega$ tion, you write:

$$
o^{3} \times 4=72
$$

Here, you are adk ing 4 threes together. Thus:

$$
3+3+3+3=12
$$

In divisicn, you write:

$$
12 \div 3=4
$$

Hore, yry are subtracting three 4 times:
$12-2-3-3-3=0$
nave given some elementary examples to give you a deeper understanding of these methods. Over the years, mathematicians have delighted in using their grasp of elementary principles to make maths easier still.

## The 'tricks' of the trade

Some people are intimidated if they have to divide a number by 5 . But they are comfortable with a round figure like 10 . In such cases, they should take advantage of the fact that 5 is half of 10 and could do it this way:

Problem: $\quad 165 \div 5=$ ?
Method: 1. $165 \times 2=330$
2. $330 \div 10=33$

Answer: $\quad 165 \div 5=33$
Similarly, if dividing by 15 poses a proble a, remember that 15 is half of 30 . Since 30 ic a round figure, it might be easier to thit in these terms:

Problem: $\quad 105 \div 15=$ ?
Method: 1. $105 \times 2,2$

$$
\text { 2. } 210 \div 30=?
$$

Answer: $\quad 105 \div 15=7$
The simple loic here is that 30 is two times 15 . So, if the a vidend is 105 , make it two times - $105 \times 2=2$. You already know that the divisor 10 n doubled is 30 . In effect, you are doubling $b \pi h$ numbers thus:
$105 \times 2) \div(15 \times 2)=210 \div 30=7$
So, $105 \div 15=7$
If you have to divide a number by 14,16 , $18,20,22$ or 24 , to simplify it to yourself, you can use numbers of lesser value such as $7,8,9$, 10,11 or 12.

Divisor 14:
Problem: $\quad 392 \div 14=$ ?
Using divisor 7:
Method:
a. $392 \div 2=196$
b. $196 \div 7=28$

Answer:

$$
392 \div 14=28
$$

Divisor 16:
Problem: $\quad 464 \div 16=$ ?
Using divisor 8:
Method: a. $464 \div 2=232$
b. $232 \div 8=29$

Answer: $\quad 464 \div 16=29$
The same method applies here:

1. $882 \div 18=441 \div 9=4$
2. $4960 \div 20=2480 \div 10=248$
3. $946 \div 22=473-1=43$
4. $1176 \div 24=509 * 12=49$

## Dividing by factorn

If you find it dificult to divide 1088 by 32, here is an easy hod. You can break 32 into its factor $101 \times 4=32$. So you can use 8 an 4 t $t$ divide 1088.
roblem: $\quad 1088 \div 32=$ ?
Method: 1. $1088 \div 8=136$

$$
\text { 2. } 136 \div 4=34
$$

Answer: $\quad 1088 \div 32=34$
Or if you are given this calculation:
Problem: $2695 \div 55=$ ?
Method 1. $5 \times 11=55$
2. $2695 \div 5=539$
3. $539 \div 11=49$

Taking the last example, you will realise how
much easier it is to do it this way rather than the way you were taught in school, which was:

$$
\begin{gathered}
55) 2695(49 \\
220 \\
\hline 495 \\
495 \\
\hline 000
\end{gathered}
$$

Of course, if you are at home usirg heschool method, don't change it. My idea is onv, to make those of you who hit a menta. bock wrien faced with seemingly large nu. nt olise that there are easier methods of don $g$ he same sum.

Let's take anoth raimple:
Problem: $892 \div 16=$ ?
Keep in minc here that to divide by numbers that are ver of $2-$ such as $4,16,18$, etc. - you have to halve only the dividend:

Ret.od. 1. $8192 \div 2=4096$
2. $4096 \div 2=2048$
3. $2048 \div 2=1024$
4. $1024 \div 2=512$

To explain : $16=2 \times 2 \times 2 \times 2$. Or $2^{4}$
So, by halving the number 8192 four times, you get the correct answer. Hence:

$$
8192 \div 16=512
$$

This technique is extremely useful when you are faced with a large number. For example:

## Problem: $32768 \div 128=$ ?

Method: $128=2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$.
Or $\quad 2^{7}$
So, you halve 32768 seven times:

$$
\begin{aligned}
1.32768 \div 2 & =16384 \\
2.16384 \div 2 & =8192 \\
3.8192 \div 2 & =4096 \\
4.4096 \div 2 & =9018 \\
5.2048 \div 1 & =10,4 \\
6 \cdot 1024 \div 2 & =512 \\
7 \cdot 52 \div 2 & =256 \\
\text { Answer: } 3.768 \div 128 & =256
\end{aligned}
$$

An imprortaht point to remember is that these method work only with certain numbers. When given rroblem, study the numbers involved. I is will help you find a quick and easy way solve it.

## Dividing by fractions

This is an extremely interesting technique since it gives you an insight into fractions and how they work in division. Obviously, it is easier to divide by whole numbers than by fractions. What do you do if you see: $360 \div 7 \frac{1}{2}=$ ?

There is a very easy way of doing it. Think. What is the link between the two numbers? What
you want is a round number with which you are comfortable. As you think, you will realise that the first step is to make $7 \frac{1}{2}$ into a round number. Try the numbers one by one. Discard those that don't suit you and take the one na . you are comfortable with.

Making $7 \frac{1}{2}$ into a round figure:

1. $7 \frac{1}{2} \times 2=15$. Discard!
2. $7 \frac{1}{2} \times 3=22.5$. Discard!
3. $7 \frac{1}{2} \times 4 \leq=0$ Take it!

To eau te the two numbers in the problem, multiple 60 by 4 too.
vil wite it down step by step:
Problem: $\quad 360 \div 7 \frac{1}{2}=$ ?
Method: a. $7 \frac{1}{2} \times 4=30$

$$
\text { b. } 360 \times 4=1440
$$

Now, c. $1440 \div 30=48$
Answer: $\quad 360 \div 7 \frac{1}{2}=48$

## Check list

There are certain simple tests that show whether a number is exactly divisible by another number - or a multiple of it. I have given them below for easy reference:

Number 2
If a number is divisible by 2 , it will and in $n$ even number or a 0 .

## Number 3

If a number is divisible by , the sum of its digits will be divisible by
Example: Is 372 div ible by 3 ?

$$
3 \times 7+2=12 .
$$

Since 12 is divisible by 3 , it means 372 is divisible by 3.

Num
If a uriber is divisible by 4 , the last two digits re divisible by 4 or are zeros.
Example: Is 3188 divisible by 4 ?

$$
88 \div 4=22
$$

So, 3188 is divisible by 4 .
Number 5
If a number is divisible by 5 , the last digit will be 5 or 0 .

Number 6
If a number is divisible by 6 , the last digit will be even and the sum of the digits divisible by 3 .

Example: Is 2076 divisible by 6?
a) It is an even number.
b) $2+0+7+6=15$
c) 15 is divisible by 3
d) So, 2076 is divisible by 6

## Number 7

Ah, this one has evaded mathemati ians There is no quick test to show if a nurnbe. is divisible by 7 .

Number 8
If a number is divisile $L y 8$, the last three digits are divisible by 8.
Example: Is 4898760 divisible by 8 ?

$$
760 \div 8=95
$$

So, 4898760 is divisible by 8 .

## Nu abor 9

If a number is divisible by 9 , the sum of its digits are divisible by 9 .
Example: Is 12122748 divisible by 9 ?

$$
\begin{aligned}
& 1+2+1+2+2+7 \\
& +4+8=27 \\
& 27 \div 9=3
\end{aligned}
$$

So, 12122748 is divisible by 9 .
Number 10
If a number is divisible by 10 , it ends with a 0 .

Number 11
This one is extremely interesting. How do you know if a number is divisible by 11?
Example: Is 58432 divisible by 11?
To work this out, add the digits in the eve? places, then those in the odd places.

The difference between the two an irs should be 11 or 0 .

Take 58432.

$$
\begin{aligned}
5+4+2 & =11 \\
8+3 & =11 \\
11-11 & =0 .
\end{aligned}
$$

So, 58432 is divisiole by 11 .
To divide b. $19,100,1000$ etc.
move decinar point one, two, three, etc. places to the left in dividend.

To joide by 0.1, 0.01, 0.001, etc.
move decimal point one, two, three, etc. places to the right in dividend.

To divide by $3 \frac{1}{3}$
multiply by 3 and divide by 10 .

To divide by $33 \frac{1}{3}$
multiply by 3 and divide by 100 .

To divide by $333 \frac{1}{3}$
multiply by 3 and divide by 1000 .

To divide by $16 \frac{2}{3}$
multiply by 6 and divide by 100 .
To divide by $12 \frac{1}{3}$
multiply by 8 and divide by 100.
To divide by $8 \frac{1}{3}$
multiply by 12 and a vide by 100 .
To divide b 25
multipl y y and divide by 100 .
Th a viae oy 50
14. iply by 2 and divide by 100 .

To divide by 125
multiply by 8 and divide by 1000 .
To divide by 10, 100, 1000, etc.
move decimal point one, two, three, etc. places to the right in multiplicand.

To divide by 0.1, 0.01, 0.001, etc.
move decimal point one, two, three, etc. places to the left in multiplicand.

To divide by 5, 50, 500, etc.
multiply by $10,100,1000$, etc.
To divide by 25,250 , etc.
multiply by 100,1000 , etc. and divide by
To divide by 125
multiply by 1000 and divide by 8 .
To divide by $33 \frac{1}{3}, 16 \frac{2}{3}, 12 \frac{1}{2}, c \frac{1}{3}, 6 \frac{1}{4}$
multiply by 100 and tivide by $3,6,8,12,16$.
The methods $\mathbf{I}$ have outlined in this chapter can be called 'icks, if you will! Why not? My purpose is o give you a 'scaffolding' to make it easier for you to divide. Such 'tricks' are used by the line or-the-party type to enthral the guests with his o. her 'mathematical prowess'! Try them ut it is all a part of the great maths-game! Instead of being intimidated, you can divide and rule!

## 12

## Shorthand of Mathe

Man is a tool-using anima without tools he is ro hins, with tools人 ${ }^{\text {re }}$ a' ${ }^{\prime}$

THOMAS CA LYIE

Mathematics yeing such an intrinsic part of my nut l make-up, I suppose I tend to take its cefriness for granted. However, a chance qem. rk from a passing acquaintance jolted me ou 0. my complacency.

A young lady who identified herself as Ashima, collared me at a party. 'I've always wanted to meet you,' she began. 'Tell me, why is mathematics in existence? Does it really serve any purpose?'
'Why do you say that?' I asked her.
'Well,' she said, 'except at a very elementary level, I don't see how it helps me in life. I mean, arithmetic is fine. But why do we have algebra? Couldn't we manage without it?' Before I could gather my thoughts, her escort appeared and took her away.

I too left the party. Her innocent remark had set me thinking and I wanted to be alone to sort out my thoughts. I thought of her words: 'Arithmetic is fine, but why algebra?'

## The oldest language

To put it simply, I think mathematics in aon al is a revolution in the communication o ideas. It is a part and process of the evolution of ankind. And if you think back, it is the aldest lnguage, the oldest philosophy in the vold. By listening to sounds, man evolved he spoken language. Every region of the wo ld had people expressing their feelings with po incy which became words. Later, language became a scientific process as we delved into its rammar. This evolution served a useful purp ose as grunts were replaced by words to commun nue better. As language flourished, it de olope nan's capacity for higher thinking.
io th early Indian sages, developing na bematics was yet another art, one more method of communication, one more way of understanding and relating to the world around them. We think that today we are in an era of specialisation. But I think our ancient sages were the original specialists. Their evolution was a constant, creative revolution. They were the original artists who explored the world and the universe through their senses. That is why they developed one more language - mathematics and gave it to the world.

It caught the imagination of the people. Like any other language, it flowered as new ideas
flowed into it. Take the English language, for instance. It has flowered and is constantly evolving because it absorbs new words from different environments. Indian words such as verandah and bungalow were assimilated into the English language only during the British

Mathematics flowered in the same ray. Addition may have helped in the anclent bar, $r$ system, enhancing a more 'blind' vebange of goods by giving them a ces air value. Then a mathematical thinker worked uthultiplication which, if you analyse it, is ath ray of addition, or a shorthand of a ditio. And just as any language has its root in ceftain conventions and traditions, so does m thematics.

Lancelot Ha rben, in Mathematics in the Making, describes the evthution of the positive or plus factors $2 \pi$ negative or minus factors. Taking the chon, bmarked the top half as the positive are 7 , yile the bottom half represented the nesaty de arena.


Perhaps tradition ruled that all maths calculations from 9 to 12 were of a positive nature, while 3 to 9 were of a negative nature. It is difficult now to figure out what functions our ancient forefathers had in their minds for each
symbol, each calculation. But then, we don't really know how every word evolved in, say, the English language. We don't even know why words spelt in the same way are pronounced differently. For example, notice the different pronunciations of 'put' and 'but'. Yet, we continue to use thena we have been taught.

## Symbols and Signs

Maths too has certain ideas in $t$ whit cannot always be explained. For eyam, le, why do two negatives produce a posit e? Perhaps the best answer is provided by Professor John T. Tate, a mathematician at Havat, when he says: 'This rule is simply a convention that is universally agreed upon bs people working with numbers because it 's useul.'

Tha bings me back to the beginning. As I said earlb, phaths was, and is, one more means of cu mntuncation, one more language added to he human repertoire. Without maths we would be writing down ideas in longhand. We would be saying: 'Two plus three multiplied by two gives ten.' But using the shorthand way makes it easier:

$$
(2+3) \times 2=10
$$

It is seeing a calculation at a glance. It is a system, a symbolic language designed because it is convenient. If it weren't for the symbols and signs, we would have reams and reams on maths until finally it would be a dead science.

Fortunately, human brilliance keeps it alive. Ashima asked: 'Why algebra?' It is a little like
asking: 'Why any other language? Why doesn't the entire world speak only one language?' Would we seriopusly want all languages, save one, to die out?

Simple equations as we know them toda, were not so simple years ago. During $\mathrm{t}_{1}$. Renaissance, algebra was a very complic ted process. For example, Pascal wrote i' as:

Trouame. i.no.chegiōtoal suo ädra.to ta. 12
Today, we know it as:

$$
x+x^{2}=2
$$

Note the difference betw a thought-process being written out in lo ghand, and the shorter, simpler way of mode $n$ algebra.

Later, a mathent ician tried to simplify algebra by declaring mat all vowels should stand for the unknown, whil the consonants should denote the known. However, the great Descartes had the Sinal w. ra Or, in this case, the final symbol! He de ia.d that the last letters of the alphabet such a. . should stand for the unknowns and the leiters at the beginning such as ' $a$ ' or ' $c$ ' should stand for knowns.

Now Descartes may have evolved this method either from a logical thought process or by some idea rooted in a certain tradition. The point is, as in any language where words have come down to us complete with their meanings, so has the symbol ' $x$ ' come to us meaning 'unknown'. We use it quite freely even in our everyday language. The $x$-factor representing the unknown is a commonly understood idea today.

## Conquering the $x$-factor

Our ancient mathematicians had given birth to an idea. The next generation of thinkers set about polishing it or distilling it, and finally, boiling down to a formula. With this formula, they could explore new areas and relationships.

We are the blessed inheritors of the rim equation, and are free to reach even wigher if we want to. When we are taught figeor gr any branch of mathematics in scho l, we ar actually participating in the great or th of mathematics. Not all of us may specian e in it, but it helps us to explore our tale ts. Inat is why a good parent exposes the chid to every possible discipline. For s'anmple, a child may have the potential to be a great tennis player, but if he is never exposed to the game, he will never know he has it inm. By exposing children to every field or nt marents and educators are giving then the -pportunity to grow.
nits simplest form, algebra is another version otarithmetic. You need not feel intimidated when you see a problem as:

Problem: $4 x+5=17$ Find $x$.
Again, it is common sense at work:
Method: $\quad$ 1. $(4 \times x)+5=17$
2. Which number added to 5 gives 17 ?
3. $12+5=17$
4. $4 \times 3=12$
5. $4 \times 3+5=17$
6. $x=3$.

Since you already know that $4 x$ means $4 \times$ $x$, the rest is easy. This may seem a long way of working it out, but our forefathers had to go through such long processes before they discovered a link and a formula was formed.

To understand an equation, look at it tro it a practical viewpoint. What is an equatio 7 It is like a scale. The two sides should 1. lance ea.h other. So, when you look at an eauction written this way:

$$
x+7=10
$$

Visualise it on a scale
Scale 1:
Scale 2:


What io required to balance the scale?
it has to be $10=10$.
So you know that by adding 3 to 7 you get 10 .

$$
3+7=10 x=3
$$

Give your intuition a chance
As you understand this fundamental principle underlying the equation, your intuition will begin to work. You won't need your visual scale to work out:

$$
x+24=34
$$

You will know that $x$ equals 10 .

It is a simple matter of subtraction. If you keep the idea of balance in your mind, your natural intuition will take over, thus allowing you to grasp each idea faster and faster, until you feel you know it intuitively. When you see a seemingly more complex equation, the 1 . question you should ask yourself is: 'What st ould I put on one side to balance the oth ste of the scale?' From this, you can go on to the more positive: 'To balance the equa tior, I nust oalance one side with the other' This w... you will be unfazed when you see:

$$
x-10=3
$$

Here, I have dotio rocely made the figure on the right side of the equation a lower number, 3 . But you kno . intuitively that $x$ is 13 , because 13 - 10 - 3 .

As he idea of balance becomes entrenched in gour $n$ in , your intuition will sharpen and your th ra eye will open wider.

What happens when you see the following problem?

$$
10 x=5 x+10
$$

How do you balance the two sides? You tackle it this way:

Then,

$$
\begin{aligned}
10 x-5 x & =5 x+10-5 x \\
5 x & =10 \\
x & =\frac{10}{5}=2
\end{aligned}
$$

So,
What may set you wondering is that there is an $x$ on both sides of the equation. But with
the idea of balancing, you know you have to 'balance one side with the other.'

With this in mind, let's go on to the next step:
Look at this equation:

$$
5 x-2=4 x+1
$$

At first sight, it appears complicated ut $t$ is not. First, see what you are comfortago- w th. You are happy with 2 and 1 because gey don't have the unknown ' $x$ ' attached in then

Now, decide which numb $r$ you would like to use to balance the on let's say, you decide on 2. What do you hav to do to balance the equation?

You have to pat 2 on both sides. So you write:

$$
5 x+2=4 x+1+2
$$

Ah, isot it already looking simpler? Now simplify it further:

$$
5 x=4 x+3
$$

G. on with one more balancing act:

$$
5 x-4 x=4 x+3-4 x
$$

That gives you: $x=3$ because all the $x$ values cancel each other out. You have solved it with this understanding. Now, if you write it down step by step without the explanations, it would read:

$$
\begin{array}{ll}
\text { Problem: } & \quad 5 x-2=4 x+1 \\
\text { Method: } & \text { 1. } \\
& 5 x-2+2=4 x+1+2 \\
& \text { 2. } \\
& 5 x=4 x+3 \\
& \text { 3. } \\
& 5 x-4 x=4 x+3-4 x \\
& x=3 .
\end{array}
$$

To give you a greater understanding of the workings of your mind, I suggest you formulate your own problem and work it out backwards. Suppose you start with:

1. $45=45$
2. $45-6=39+6$
3. $(15 \times 3)-6=(13 \times 3)+6$
4. $15 x-6=13 x+6$.

If you toy around with rurabes this way, you will get to know equations, line the back of your hand!

With the next probi m, keep in mind the idea of balancing.

Problem: Thare two numbers with the difference of 3 'etween them. The difference of their suares is 51 . What are the numbers?
Metiod. One number is $x$
The other number is $y$
3. $x-y=3$
4. $x^{2}-y^{2}=51$
5. $\frac{x^{2}-y^{2}}{x-y}=\frac{51}{3}=17$
6. That means, $x+y=17$.

Note: $x^{2}-y^{2}=(x-y)(x+y)$
7. $x-y=3$. So, $17+3=20$
8. Since there are two numbers,

$$
2 x=20
$$

9. So, $x=\frac{20}{2}=10$
10. 

$y=7$.

As your intuition works on balancing every equation you come across, you will find that even the most-complicated problem can be solved simply. In other words, you have conquered the a-factor!

## 13

## Mathability: Human Face

 of MathermaticsYears a ${ }^{20}$, when $I$ was young and inexpe iled, if someone had asked me, 'What is -a/glus?' I would probably have bored then bytang like a mathematician! But -xpery a subject to something the other person anderstands will the listener get the grasp of it. The comparison may not be a hundred per cent accurate, but it is sufficient to light that spark of interest.

If I were to tell you that calculus is about 'optimums' or 'rate of change', you would switch off mentally. But I remember telling somebody: 'Calculus is to maths what astrology is to astronomy.' Instantly, he was captivated. The description was not accurate, but it caught his interest. In fact, he held up his hand and
exclaimed: 'Wait! Don't tell me. You mean that calculus finds relationships and analyses them?'
'Bingo!' I replied.

## Director of maths

Since calculus has never been marketed, it is viewed with awe, or with indifference, not at all! Yet, it is the extremely vibrant, vely Human face of maths. In calculus, you don h h.e po deal with numbers in themselves bu 4 th illustrations, such as shapes and gra, Its. It has an inbuilt technique. For exampe, a s riptwriter may write an excellent script, con plete with a tight plot, strong characters, aid ffective dialogues. But that script will nôt buppreciated unless a director can bring it to life on the screen.

The dir cto thas the intuition and ability to fine-tuns the script, highlight small details thus making the-m dramatic. He does this with a keen so st of awareness of what would appeal to the Fub.ic. In other words, he is able to perceive the reationship between the written script, the way it will unfold on the screen, and audience appeal. That is what calculus is all about. It is the director of maths. It analyses and tunes relationships.

Take the hypothetical career of an Indian film actress. She is excellent at emoting, and is keen to take on varied roles. She gets three roles one after another, each different from the other.

The first role is of Sita, a beautiful and virtuous young woman. She will do anything for her husband, even walk through fire if he asked her to.

The second role is of Gita, a modern-day Rani of Jhansi. A fighter whose eyes flash fire and brimstone.

The third role is of Rita, a playgirl. She beguiles men, plays with their emotions, and discards ther when she gets bored with them.

You will notice that I have taken theo es with extreme characteristics. To the actress, each role is a challenge as it calls uponather acting prowess. With her immense ta 'en ' she is poised for superstardom. But, in pratic\% does it work that way? If you we to plo a graph of her career, you would expect it to zoom steadily upwards. This is how her graph ought to look:


But in reality, this may not happen. Why? Because we have based our calculations purely on her talent. With each role she has excelled herself. But this is where the audience reaction sets in. So her graph will read:


Why is there this nexpe ted downtrend? The key lies with the audie ce. They revered her as the pure Sita. They ado red her as the gutsy Gita. But they hated no as the sly Rita.

In perce ving his relationship lies the science and art 1 ulus. This is where terms like 'infinite $2 \%$ 'limits' come into calculus.

## at uns and the 'real world'

By now, I am sure you are beginning to understand this fascinating subject.

Some months ago I read an article about Asian workers. Traditionally, Asians have proved to be hard-working. They work twice or ten times as hard as non-Asians, at lower wages. Earlier, stricken by extreme poverty, they were even willing to work double shifts. Since they were paid by the hour, the more hours they worked, the more they earned. The word 'leisure' did not exist in their vocabulary. But with the improvement of
their economic situation, came the gradual realisation that there was more to life than work. They discovered a life of leisure, of spending time with their families. So now, while they were willing to put in twelve hours, they were not willing to put in eighteen. They had reached int 'limits'. After a certain point, they valued Ne ure more than the extra money they could arm y working extra hours. Overtime was no onger that attractive.

This is where words like op:imum level', 'limits' come in handy. is is where calculus takes over. The gropt read:

No. of Asians
willing to work oy me


If you were to extend the graph to 20 or 24 hours, the curve would go lower. Such a graph would prove invaluable to an employer. From it he would be able to ascertain a pattern relating to the real world. The 'real world' perception is important because his paperwork would obviously show him the following:

1. His product has a vast, limitless market.
2. If his workers worked round the clock, he'd be able to get maximum production and maximum profits.
But, in practice, this may not work. His workers must be willing to work 24 hours, or he will have to employ more workers for $\mathrm{Mr}_{\mathrm{g}}$ ht shifts. He will realise that such a situation hits a peak, but the human element begin, to sov ly lower it. The slope of the curve $w \cdot 11 \mathrm{~b}$ - p him to work out an optimum sit/, ation, bus snabling him to chart out the most effi ie.. and effective system.

You may have no iced Lat both examples I have cited have a linl. L th show the real world, the human element the $t$ runs so strongly in the best written pojec report. That is why I say, calculus is the brant, human face of maths. It has the in hent power to perceive and analyse a situat on uninformed employer may think tha the only incentive his worker needs is more m. ne, : But calculus will show him the human in eds. Ironic, isn't it, that maths shows the human side to a human being?

Similarly, the actress may take on roles that are similar to the roles of Sita and Gita and turn down the Rita roles. You may mourn the fact that she has not explored her talent to the fullest, but the actress must weigh her inner satisfaction against box office success. Calculus makes her aware of the swings. It is up to her to choose.

This also holds true for, say, a shopkeeper. If he sets up shop in a competitive area, he will have to work out the optimum levels of sales to
get a realistic picture. By careful study, he may find that he can succeed if he keeps his shop open later than the rest, or keeps it open on Sundays when others are closed. Perhaps, if he adds on a home-delivery service, he will stan a better chance of success. A maths-model claims, the situation and enables him to find the $p$ aks and the plateaus.

## Finding relationships

Calculus also plays a. 1 P prant role in architecture. Erecting A $n$ effe tive dam or building has its roots in calculus The architect works out his blueprint with ait derstanding of the precise relationship betwen forces. In effect, calculus catches the fowce. working on any given situation. Then it orga nise; those forces. In the final analysis of these forces lies the key to the solution. The forces rar oe time, space, motion . . . .
C. Vculus helps in finding the area of different s. ay es. Most of us know that a square can be $2 \mathrm{~cm} . \times 2 \mathrm{~cm} .=4 \mathrm{sq} . \mathrm{cm}$. or a rectangle $2 \mathrm{~cm} . \times 4 \mathrm{~cm} .=8 \mathrm{sq} . \mathrm{cm}$. Here, we know that length $\times$ breadth $=$ square area. But how do you find the area of a strange shape that does not conform to these set patterns? Take a swimming pool shaped like a piano, or the continent of Australia. Neither of them is a square, a rectangle, a triangle, or a circle. But by dividing the shape into several squares or rectangles, you can calculate its area. I won't go into technical details or the methods because I am not conducting a course in calculus through this book! What I'd
like to do is to show you not just the usefulness or the relevance but also the human face of the subject.

Calculus has its own terminology which often makes it seem a difficult subject to tackle. Frr example, curved shapes are called 'functions arrive at the human element, you will have to learn the science of it, grasp the coricept, b it 'once you have absorbed the basic ile or what calculus does and can do, you wil nivenger look on it with awe or indiffere cu Simply put, calculus is finding relatio $\mathrm{shn}_{1}$, s . If you meet a new cousin, wouldn's you e interested in how he or she is related to vod? Similarly, calculus is the family tree of naths. In its branches lie the fruits of efficioly and effectiveness.

## 14

## Eccentricities of Maths

Genius means little more than the , cury of perceiving in an unkutial sale william james

As I write thy chapter, I presume you have been e. d. $\boldsymbol{r}$...e book i. c...o...lo oc. 1 ...... - chapwronchapter. You may or may not agree with ev rytbing I have said, but I am sure that vo low view maths with more understanding ar d less fear.

In any field there is a degree of self-analysis. I know a writer who is so charmed by words that if she doesn't write her quota for the day, she feels unfulfilled. She says: 'Words turn me on. I see every day as a new birth in which I gather a bouquet of fresh experiences and thoughts.' Her ultimate nirvana is to write it all down by analysing what she has gathered and including it in her articles or stories.

## Self-analysis can be fulfilling

I also know of a brilliant Indian classical singer. She is an interesting study. She feels she is beautiful only when she sings. When she was interviewea by a magazine, she looked forward to sonil $\alpha$ herself in print. While the interview was 0 , the photographer kept circling her, photogranh.ig her in various candid poses. When the featime was finally printed, she was devastatei ine elt she looked ugly in the picture. I should have forbidden it. I shoula ta e asked them to photograph me at a solo, erformance.' She did not mean that she wou'd look better because she was dressed for the how or had make-up on. What she meart was that being a true lover of music, it was nly while she was singing that she achieved true beauty. This was when her very soul shont through in all its glory and made her bealat And this was the moment for the Ho. grapner to capture her on film.

I have captured such moments and thoughts and shared them with you to show you the wonderful art of self-analysis. Such an exercise carried out in moderation is extremely fulfilling. There is a romance to it and a sense of satisfaction. I want you to reach out and capture that warm, glowing, indescribable feeling and use it for maths.

Some psychologists believe that by delving into your past and examining, step by step, why you dislike maths, you will understand yourself and begin to like maths better.

I am the kind of person, however, who prefers to look ahead rather than back. Perhaps you can or cannot wipe out the past completely from your mind but, believe me, the best way to forget the past is to put it firmly behind you and look to the wonderful present and the glorious fu u. You can do it if you want to.

Let me assume that you have a esmortatie lifestyle. It is immaterial whether you ve a arried or divorced or whatever. Yot a fe friv content with yourself. You have almos everything you need to meet your socia' $p$ ychological, and bodily requirements. Curionity or some other emotion has made ye p. ck up this book. So now, let us look ahead.

Do you want to do maths? But maths for what? In this question lies the core of your selfanalysis Marry people think that they should do matho on's if they are planning to become mati mativans, engineers, or technocrats. That
ke saying you should exercise daily only if you want to take up a sport professionally or become an athlete or a gymnast.

By being able to look at maths straight in the eye, you are able to take more control of yourself and of your life. It is amazing how much a small change in the pattern of life can do for you. Somehow, a little shift brings out the fighting spirit in you. As you learn to tackle one more area, you discover a 'plus' in you. In doing so, you have a great feeling of relief, of freedom, of having conquered something.

## Demand the best from yourself

The best thing about starting something new is that your'age doesn't matter. There is no time limit. You are not in competition with anybody. What is even better news is that since you are an adult, you have a rich store of experien e, s? you become a better mathematician because of what you have seen, heard, and done. in ur abili.y to analyse is much sharper than it wao won you were an innocent fledgling A : cho l.

With absolutely no pesc re you, you can relearn maths by chonsing the method that suits you best. There is no ac ult or authoritarian figure standing over you, $1 \cdot m_{1}$ nding the impossible. There is no set prodediare, no regimen involved.

If you are © mfortable with words, you can work on it through sentences. If you find you enjoy nt mers, you can discard the words. If you are a suas ser you can enjoy yourself by working on $g$ apis or other diagrams.
other words, what you are doing today is not learning or memorising a subject because it has to be done. Instead, you are taking on maths because you want to. You are seeing it as one more skill to be developed. Yes, a skill - such as playing table tennis or riding a bicycle or driving a car.

You can acquire and sharpen this skill in various ways. You can solve maths puzzles as you would a crossword puzzle. You can learn little 'tricks' to try on your friends. You can try out all those 'shortcut' methods that you couldn't in school. Oh, the potential is enormous!

## Soaring high

Look on this phase of your adult life as: All the things I couldn't do in maths when I was in school that I can do now. You are now free to enjoy the peculiarities or eccentricities that are so typic 1 of Old Man Maths. For example, take the numb 8 and see what it does:


You can imbibe th, mystique of the number 9. Look at the mutiplication tables:
$9 \times 1=9$
$9 \times 2=18$
$9 \times 3=27$
$9 \times 4=36$
$9 \times 5=45$

Now, add up the sum of the digits in each answer:

$$
\begin{aligned}
& 9+0=9 \\
& 1+8=9 \\
& 2+7=9 \\
& 3+6=9 \\
& 4+5=9
\end{aligned}
$$

The sum is always 9 !
Or take a number like 87594. Reverse the order of the digits: 49578. Subtract the lesser number from the greater one:

|  | -49578 |
| ---: | :--- |
|  | $=38016$ |

Now, add the sum of the digits in the remainder:

$$
3+8+0+1+6=18
$$

Again, add the sum of the digits in the amsw y:

$$
1+8=9
$$

So, it is back to King Ne
It is a stimulating, $a$ or. m y I am a great believer in spiritual armo v. I think that when you are absorbed in maths - which is the structure that our inn erse is built upon - you are in touch witr urself. You are in touch with your instincts, our heartbeats. And just as the singer felt hat) sne was beautiful when she was singing, so it is with maths. It is when your spirit is stim la -d hat it soars on the wings of harmony.

## 15

## Mathability: That

## Extra Step

If you want to do sqmang, do it!

Why bave ywritten a book on ow to li ii at maths-alienness and encourage more p op to take up maths? It is because I awant to avoken within every reader the sleeping gh. ni that slumbers due to ignorance or fear. That seoping giant is 'Inspiration'. An unexplained, powerful, energetic force that, once awakened, gives you a great gift. The gift of willingness. The gift of action. The strong upsurge that makes you take that extra step.

## Be an achiever

That extra step you take will make you an achiever. What is an achiever? A millionaire? A TV star? A politician? He is much more than that. An achiever is one who has accomplished something. A person who has attained his
objectives. One who has overcome his fear. He does not have to be famous to be an achiever.

Every achiever - known or unknown - in the world is a successful person because he or she has taken that extra step. That is what I war t you to do. You may already be a success in you r chosen field. But if you overcome you fea. of maths, you will be taking that extra p whiln will help you reap greater benefits anibing you joys that have so far eludea ycu.

Take the example of $f$ no He who takes that extra step and sows o. e riore seed, pleases the great mathemafician - Nature. She rewards him by multiplying th t one little extra seed into several hundred rains. The farmer has one more achievement to his credit. He is blessed with a richer crop bedause he had the foresight, the willingress, tme energy to take that extra step.

## The enefirs of maths

T. evenefits of that extra step are enormous. Let is examine them at a practical level and enumerate them. What do you gain by taking on maths?

- It makes you regard yourself with greater respect and in turn invokes respect from those around you.
- It provides you with greater opportunities for self-exploration and self-advancement.
- It compensates you by showing you another way of increasing your earning capacity.
- It makes you self-sufficient because you can plan your investments without any qualms.
- It leads you to develop your mind by adding one more accomplishment to your brain's repertoire.
- It protects you from job-retrenchmen ir difficult times, because you have in a skill that might induce your emplower to retain you.
- It enhances your job prost ac because you possess that importars, s il,
- It makes you mor awa more alert, more keen because is is ${ }^{\circ}$ constant source of inspiration.
- It develop: innuative and enterprise within you, thanks.o. your new-found confidence. It \&ives vihers confidence in you and your at int
banishes the old habit of procrastination that in the past prevented you from doing certain important things - such as balancing your chequebook.
- It gives you a purpose, an aim, a focus that insures you against restlessness.
- And finally, it makes you a recognised citizen of a wonderful new world - a world which follows the principles of the law of increasing returns. As I have said earlier, you have nothing to lose and everything to gain by learning maths.

Fling away those old, rusty fears.
Let your clean metal shine through.
Let your" inspirational juices course through you. Maths is your friend.
Maths is your instrument to a richer life.
Recognise the loyal friend who will never $h$ you down.
Give yourself a chance by giving maths a chan Maths has the answers, the solvith ns.
Take maths on and become an achiove
Take that vital extra step!


## 16

## Myths About Maths

Who doesn't njoy a bit of gossip? Film and poli. c. personalities are its first target almost a yhere in the world. Gossip adds spice to lis. Th ere' something deliciously wicked about it. Ma 'is has its share of gossip too! Take a piece ot odvice from me: enjoy it, then forget it. After , H, as Pascal put it so well, it boils down to two plus two making five!

Let's have a cosy gossip-session about maths, and then see what the real truth is behind the gossip.

Myth: Women mathematicians are masculine.
Truth: Who says so? Do you think an intimate knowledge of the magic of numbers makes women suddenly sprout bristles on their cheeks? No way. In fact, as
far back as the 60s, American researchers came up with the startling revelation that women mathematicians were 'significantly more feminine'!
Myth: Non-mathematical males are sissy ard feminine.

Truth: Rubbish! Is there any medial cof for this? But this senseless allegation makes the poor non-motive rale feel as if he has gone in or sor-change operation! He is acha ned to admit that the only 'pi' he, list es is Mom's Apple Pie!

Myth: When it co ne, to maths, you have to lock in your intuition and let pure locic take over.
Truth: Tsk. tsk! Without intuition where would logic be? For example, you've w- tched a murderer commit a crime; then he tries to convince you in a logical way that you never really saw it. Despite his persuasive arguments, will you be convinced? Of course not! Heed the wise words of one of the greatest mathematicians the world has ever produced: 'To these elementary laws (of physics) there leads no logical path, but only intuition supported by being sympathetically in touch with experience.' It was Einstein himself who came to this conclusion!

Words like 'rational', 'precise', 'logical' have given poor Old Man Maths a bad name! Yet mathematical history points towards intuition. Newton intuitively knew calculus existed somewhere on the fringes of the consciousness of mathematics. But he couldn't devise the losen steps required to give flesh and blood to the skeleton of his idea.

In mathematics, intuition is the parent of invention. Logic follows like $A$ well- raned dog. In fact, maths does not have its roots in logic at all. It is rooted in idtos thake life more practical, more orgarin ed. 'hose ideas are born out of observation and rtution. René Descartes, who has been given th awesome title of 'Father of Modern Matrmatics', was one day idly watching a fl C awl on the ceiling of his room. His intuitive mihd caught an idea that made him chart the path of the fly in maths-lingo. And analy the 6 coonetry was born! It depends on what and $w$ here your mind is tuned to for that moment. the Sottish king Robert Bruce deduced an entirely diverent philosophy from watching the spider!

An industrialist who wants to set up a new plant may ask for data, profit and loss projections, and so on, from his marketing men. They may come up with a picture that proves that the new project is going to be a financial disaster. They may advise him through this logical process not to go ahead with it. But if his intuition tells him otherwise, he will use their projections to evaluate the risk factors and still go ahead. The eventual success of the project is not due to logic but to his intuition!

Each one of us has this special mathematical intuition within us. We have to learn to listen to it.

Myth: Approximate is for the birds, exactness is for mathematicians.
Truth: Well, well, well! I'd rather be a bird in that case! Of course, exact ont h do help, but only a icientific laboratory levels. Take $n$ at trom the fact that 'approxima'e is what we live by in any fiela Vom you tot up your grocery bit you noedn't do it to the last paisa or rent. A once-over, just to satisfy you seí that it's not wide off the imark, is enough. Even mathsorien ted knowledge hinges on the pproximate. The distance from the earth to the sun is approximately 15),000,000 kilometres. The surface area of the globe is approximately 500 million kilometres.

It is only in school that you are taught to get exact answers at the simple arithmetic level. This is because you are being taught a skill and should learn to be as perfect as possible at that point of time. But later, there is no real value or satisfaction in being accurate to the last digit. For example, in a project report a figure such as 3.65 is rounded off to 3.7 , and so on. So, it all depends on the circumstances. Even a doctor says that if his patient is over fifty-five years of age, a blood pressure level of about 140 to 150 is fine.

Myth: If you count on your fingers, you are a philistine!
Truth: Pooh! For some reason, counting on the fingers is not seen as 'elegant'. Yet, many do it under the table, behird their backs, and so on. What paren and teachers have to understand s that there's nothing writy with counting on the fingers. What is the abacus after all? It is a sligntly more advanced way of roling on the fingers. Strange'd er ough, the constant use of a colculatir is not as frowned uport as ou ting on the fingers. A calculator inly makes you lazy and robs you of the ability to calculate on your own. Counting on the fingers 'oes $n^{\prime} \mathrm{t}$. And as I've said earlier, maths began at our fingertips, so why the new inhibition?
$N_{1}$, th. The faster you solve a new problem, the smarter you are.
Truth: Yes, if you are planning to compete against Carl Lewis in a 100 m race! Mathematicians are no more speed merchants than are writers. A good writer can "take years to write a novel. Here, speed is not essential. So also in maths. Speed proves nothing, absolutely nothing. What counts here is experience and practice. A classical singer makes her performance look so fluid, so effortless, but it is hard work,
practice, and experience that make this possible. So is it with mathematicians. Pen and paper are required. The time mathematicians take depends on their experience - whether they've solved similar problems before. If there 10 variation, their intuition too come in handy. And don't forget thar spec al person - Lady Luck!

So don't get discouraged a no throw ap your hands if you come acrow reblem that looks tough. Just pick up the p n, think, and start!

Myth: You have o pove the memory of an elephat to be a good mathematician. If th. ${ }^{+}$were so, elephants would be good mathematicians! Think of it this way: when you begin to learn the English language, you have to first nemorise the alphabet, but as you go on, it becomes a part of you. This includes grammar and vocabulary. Similarly, with maths you should allow the symbols and the practice of them to become a part of you. After all, doesn't 'two plus two equals four' roll easily off your tongue?

What is maths? It is the same as any language. You have to understand the concepts in it just as you have to understand the grammar in a language. Grammar has rules; and maths has formulae.

Unfortunately, from an early age we are taught to rattle off multiplication tables like parrots. Few children grasp the concept of tables - that $2 \times 2=4$ is the same as $2+2=4$, or that $3 \times 3=9$ is the same as $3+3+3=9$.

I have an amusing incident to relate whin regard to tables. About to knock on the doc. of my friend's home, I noticed that it ins slighiy ajar.

Planning to surprise her, I vushe. it open fully, but stopped when Ahord this conversation between her and her ten rea-old son:
'Where are you go ing?'
'To play.'
'Have you learst your sixteen times table?'
'Yes, Mamm.
'All $\mathrm{rr}_{2}$ h. Recite them to me. I want to hear them.
an't recite them later?'
No. Right now.'
Pause. Then the boy's voice started in that sing-song manner. It went: 'Sixteen-ones-aresixteen! Sixteen-twos-are-thirty! Sixteen-threes-are-forty-five!'

He paused to take a breath and my friend said, 'Very good! Go on!'

Unable to suppress my laughter, I walked in. Looking sheepish, the boy stopped. Obviously, he neither had the interest nor the understanding of the tables. He was reciting the fifteen times table to his mother who didn't know any better!!!

Such incidents bring home the fact that people think tables and other mathematical concepts are to be memorised, not understood. Yet, in any field only by understanding do you remember Even the lyrics of a song are remembered bettr r if you have understood the meaning of the worms. That is why some people say they are wrddeaf'.

In maths, a proper grasp of tre Jasics is essential. This grasp helps a. p roblen become increasingly complex. Meming a formula won't help if you hav n't understood the application of it.

Myth: There's no flash of inspiration in maths. . i s all perspiration!
Truth: if tha were so, why did Archimedes jum out of his bath, yelling the now famous word 'Eureka'? That's because be had a flash of inspiration. Both these qualities are required to solve maths problems.
Myth: You either have the ability to do maths or you don't. Mathematicians are born, not made.
Truth: For mathematicians, what a lovely image to have! If this were true, they would be on cloud nine, with people looking at them in awe for their special 'inborn' abilities! But the truth is that one is not born a mathematician. It is an acquired skill, a skill that can be taught to everyone. But the false idea
that mathematicians are born not made, raises mental barriers for most people. They tell themselves: 'I can't do it because I was not born with a maths brain.' As they go on repeatirg it, they undermine their own se confidence.

## Believe in Yourself

Maths is exactly like a sport. I a sportsperson feels she can't better the wore rord, she'll never do it because she hat not reared herself for it. Negative emotions stop her from even trying. Oh, you may see her fun ing on the track, but her heart won't be in it So also with maths. Believe in yourself, heh ve you can do it and you will be able to to lt . If it comes to that, I'd say we are all bon to maths, but we don't know it.

No \% thet you have read about the myths w. at for it is, I hope you will be able to discard a y false notions you may have about maths. It's great companion and has all the requisites of life in it - work, fun and games, and even myth!

## Touch the future!


"We carry within us the seen of the genius we seek without. Each one of us possesses a mathematical brain. Án 1 grained power, a creative energy, a vea er problem solving capacity than we probably give ourself credit for With a botre shderstanding, I am sure you can influe ne yourself and your child to take maths on. - is agreat subject. Dont knock it. It is your and vour child's competitive edge. It is the future.".


